NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

A TIME DOMAIN APPROACH TO SENSITIVITY ANALYSIS OF DIRECT DETECTION OPTICAL FDMA NETWORKS WITH OOK MODULATION

by

John Anthony Studer

March 1995

Thesis Advisor: Tri T. Ha

Approved for public release; distribution is unlimited

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	March 1995	Master's Thesis

4. TITLE AND SUBTITLE
A TIME DOMAIN APPROACH TO SENSITIVITY ANALYSIS OF DIRECT DETECTION OPTICAL FDMA NETWORKS WITH OOK MODULATION

5. FUNDING NUMBERS

6. AUTHOR(S)
John A. Studer

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Postgraduate School
Monterey, CA 93943-5000

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES
The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the United States Government.

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

We derive the closed form expression for the probability of bit error in direct detection, dense wavelength division multiplexed (WDM) fiber optic systems employing OOK as the modulation technique and single-cavity Fabry-Perot (FP) filters in the receiver as demultiplexers. The expression is derived in the time domain using the impulse response of the single-cavity Fabry-Perot filter and the complex baseband equivalent of the received dense WDM optical signal. The two are convolved to produce the FP filtered output signal $s(t)$. We then integrate $\mathcal{R}|s(t)|^2$ over one bit period, where $\mathcal{R}$ is the responsivity (A/W) of the photodetector following the FP filter in the receiver structure. This integral is the deterministic X of the decision variable Y where $Y = X + N$. N is the postdetection thermal noise (amplifier generated), a zero mean Gaussian random variable with variance N_0T where N_0 is the noise current spectral density (A^2/Hz). Both X and N are combined into an expression for probability of bit error. A limited case of the complete model is assumed, and probability of bit error graphs are generated.

14. SUBJECT TERMS
direct detection optical FDMA, optical Wavelength Division Multiplexing (WDM)

15. NUMBER OF PAGES
206

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102

Approved for public release; distribution is unlimited

A TIME DOMAIN APPROACH TO SENSITIVITY ANALYSIS OF DIRECT DETECTION OPTICAL FDMA NETWORKS WITH OOK MODULATION

by

John Anthony Studer
Captain, United States Army
B.S. Biomedical Engineering, Boston University, 1984

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL

March 1995

Author: [redacted]
John Anthony Studer

Approved by: [redacted]
Tri T. Ha, Thesis Advisor

[redacted]
Randy L. Borchardt, Thesis Co-Advisor

[redacted]
Michael A. Morgan, Chairman, Department of Electrical
and Computer Engineering

ABSTRACT

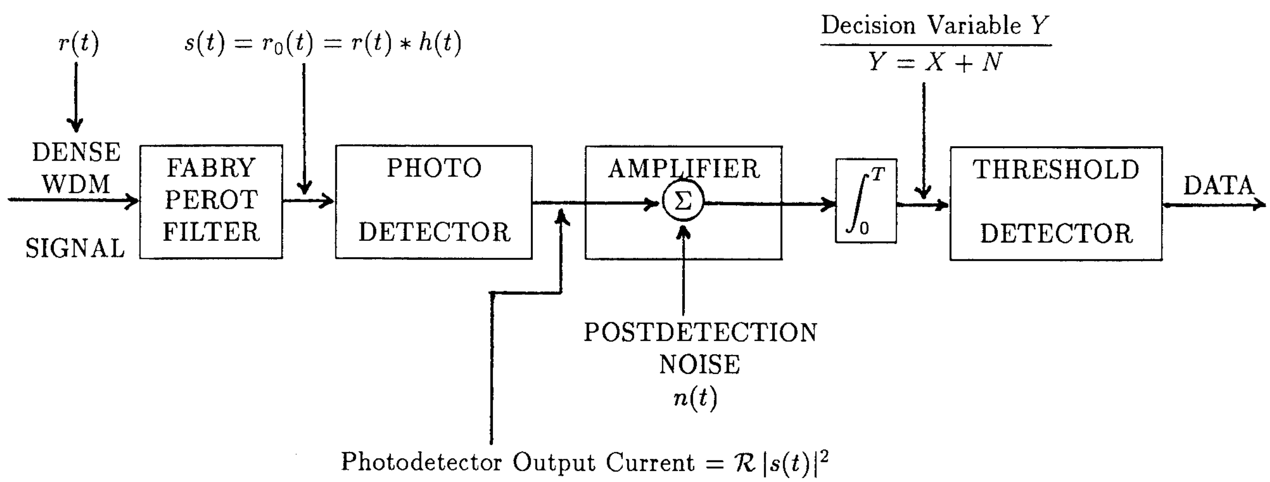
We derive the closed form expression for the probability of bit error in direct detection, dense wavelength division multiplexed (WDM) fiber optic systems employing OOK as the modulation technique and single-cavity Fabry-Perot (FP) filters in the receiver as demultiplexers. The expression is derived in the time domain using the impulse response of the single-cavity Fabry-Perot filter and the complex baseband equivalent of the received dense WDM optical signal. The two are convolved to produce the FP filtered output signal $s(t)$. We then integrate $\mathcal{R}|s(t)|^2$ over one bit period, where $\mathcal{R}$ is the responsivity (A/W) of the photodetector following the FP filter in the receiver structure. This integral is the deterministic X of the decision variable Y where $Y = X + N$. N is the postdetection thermal noise (amplifier generated), a zero mean Gaussian random variable with variance $N_0 T$ where N_0 is the noise current spectral density (A^2/Hz). Both X and N are combined into an expression for probability of bit error. A limited case of the complete model is assumed, and probability of bit error graphs are generated.

TABLE OF CONTENTS

LIST OF FIGURES

I. INTRODUCTION

Direct detection optical frequency division multiple access (FDMA) networks are increasingly becoming an attractive alternative to coherent optical FDMA networks [1]. One of the primary reasons is that noncoherent systems do not require expensive synchronization circuitry for proper operation. Also, present optical filter technology allows designers to closely pack the channels in frequency, resulting in dense wavelength division multiplexed (WDM) systems that can provide aggregate bit rates of many terabits per second (1T b/sec = 10^{12} bits/sec) [2]. The possible uses of dense WDM systems are many (local area networks, undersea surveillance, etc.), but one can easily see the enormous economic benefit and importance of being able to transmit aggregate bit rates of terabits per second on a single fiber without the economic burden of expensive synchronization circuitry imposed by a coherent system. In this thesis, we derive the complete closed form expression in the time domain for the probability of bit error of dense WDM systems employing OOK as the modulation technique and single-cavity Fabry-Perot (FP) filters as channel demultiplexers. In our derivation, we make no simplifying mathematical assumptions and use the impulse response of the single-cavity Fabry-Perot filter, which is an infinite sum of delayed impulses whose intensities decrease geometrically. The results of our work are presented in Chapters II, III, and IV. A detailed derivation of the deterministic signal component of the decision variable appearing at the output of the integrator of the Channel 0 (Channel of Interest) receiver appears in Appendix A. See Fig. 1 for a sketch of this receiver. Appendix B shows how the complete model is reduced for a limited case and how probability of bit error calculations are made for this limited case. Appendix C shows the strategy and programs used to obtain the probability of bit error for four values of free spectral range-bit period product and a given range of signal-to-noise ratios. It is interesting to note at this point that, although the model presented in this thesis is strictly derived, complete, and done without approximation, its major weakness is that it is computationally very intensive. In fact, it takes several months of computer time on several SPARC-10 workstations working simultaneously to generate a single graph of the probability of bit error for this system. At the end of the thesis we conclude that, although our model is mathematically correct, a discrete time approach to the problem is probably a more efficient investigative tool in analyzing the performance of dense WDM fiber optic networks.



$$X = \mathcal{R}\int_0^T |s(t)|^2 dt \qquad N = \int_0^T n(t)dt$$

Figure 1: Channel 0 OOK receiver structure.

II. ANALYSIS

An OOK, dense WDM system utilizing single-cavity Fabry-Perot (FP) optical filters as demultiplexers consists of $M+1$ transmitters (fixed wavelength lasers) connected over a fiber link to $M+1$ receivers. Each receiver contains a frequency selective Fabry-Perot filter to demultiplex one of the $M+1$ channels. In our derivation of the closed form expression for the probability of bit error for this system, M is the even number of adjacent channels symmetrically placed in frequency around the Channel of Interest (Channel 0) whose carrier is transmitted on an arbitrary wavelength λ_a. The Channel 0 receiver is shown in Fig. 1. The dense WDM optical signal is received by the Fabry-Perot filter tuned to λ_a, which allows the Channel 0 data signal to pass and rejects signals in the adjacent channels. A photodetector then converts the filtered optical signal to a current. The photodetector has an arbitrary responsivity $\mathcal{R}$ (A/W). The current is then amplified by a low-noise amplifier which adds a postdetection thermal noise $n(t)$ with two-sided current spectral density N_0 (A^2/Hz). Both the signal and noise currents are now passed to an integrator (low pass filter). The output of the integrator is the decision variable Y, which is compared to a threshold V_T to determine whether an "0" or "1" was sent.

A. THE DENSE WDM SIGNAL AT THE INPUT OF THE FABRY-PEROT FILTER: $r(t)$

Again we note that Channel 0 is our channel of interest. We can write the expression for the Channel 0 data signal in the complex baseband as

$$b_0(t) = \sum_{i=-L_0}^{0} b_{0,i} p_T(t - iT) \tag{1}$$

where T is the data bit period (s), and $b_{0,i}$ is the bit in Channel 0 during the time period $[iT, (i+1)T]$. Note that $b_{0,i} \in \{0, 1\}$. L_0 is a positive integer which represents the number of bits in Channel 0 that are trailing the detected bit or bit of interest $b_{0,0}$. $p_T(t)$ is the rectangular pulse function defined as

$$p_T(t) = \left\{ \begin{array}{ll} 1, & 0 < t < T \\ 0, & \text{otherwise} \end{array} \right\} \tag{2}$$

Channel k is any arbitrary adjacent channel. There are M adjacent channels placed symmetrically in frequency around Channel 0. M is an even integer. The indices for k are as follows $k = -M/2, \cdots, -1, 1, \cdots, +M/2$. We may now write the complex baseband expression for the k^{th} channel data signal

$$b_k(t) = \sum_{\ell=-L}^{0} b_{k,\ell} e^{j\omega_k t} p_T(t - \ell T) \tag{3}$$

where ω_k is the radian frequency spacing between Channel 0 and Channel k and $\omega_k = -\omega_{-k}$. We have already defined the pulse function $p_T(t)$ in Eq. (2) above. L is a positive integer which represents the number of bits in Channel k which trail the bit in Channel k that is the 0^{th} bit, $b_{k,0}$. We now note that $b_{k,\ell}$ is the bit in Channel k during the time period $[\ell T, (\ell + 1)T]$ and that $b_{k,\ell} \in \{0, e^{j\phi_k}\}$, where ϕ_k is Channel k's phase offset from Channel 0. ϕ_k is assumed to be a uniformly distributed random variable between 0 and 2π ($\phi_k \sim U[0, 2\pi]$). We can now use Eqs. (1) and (3) to write the complex baseband equivalent dense WDM received optical signal which appears at the input of the Channel 0 Fabry-Perot filter (see Fig. 1)

$$r(t) = \sqrt{P}\, b_0(t) + \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sqrt{P}\, b_k(t) \tag{4}$$

and

$$r(t) = \sqrt{P}\left(\sum_{i=-L_0}^{0} b_{0,i} p_T(t - iT) + \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{0} b_{k,\ell} e^{j\omega_k t} p_T(t - \ell T) \right) \tag{5}$$

where P is the received optical power.

B. THE OPTICAL OUTPUT OF THE FABRY-PEROT FILTER:

$s(t) = r_0(t)$

To arrive at the optical output of the Fabry-Perot filter $s(t)$, we must convolve the input signal $r(t)$ with the impulse response $h(t)$ of the filter

$$s(t) = r_0(t) = r(t) * h(t) = h(t) * r(t) \tag{6}$$

First, however, we need the expression for $h(t)$. The Fabry-Perot filter is a causal, linear, time-invariant (LTI) system. It can be shown [1] that

$$h(t) = (1-\rho)\sum_{g=0}^{\infty} \rho^g \delta\left(t - \frac{g}{\beta}\right) \tag{7}$$

where $\delta(t)$ is the *Dirac Delta Function* defined in two parts

$$\delta(t) = \left\{ \begin{array}{ll} 0, & t < 0 \\ 0, & t > 0 \end{array} \right\} \tag{8a}$$

and

$$\int_{0^-}^{0^+} \delta(t)dt = 1 \tag{8b}$$

Looking at Eq. (7), we note that ρ is the power reflectivity of the single-cavity Fabry-Perot filter and β is the filter's free spectral range (Hz). Performing the convolution operation yields the following (see Appendix A for details)

$$\begin{aligned} s(t) \;=\; & r_0(t) = \underbrace{\sqrt{P}\,(1-\rho)}_{K}\Bigg[\underbrace{\sum_{g=0}^{\infty} \rho^g b_{0,0} p_T\left(t - \frac{g}{\beta}\right)}_{s_B(t)} \\ & + \underbrace{\sum_{i=-L_0}^{-1}\sum_{g=0}^{\infty} \rho^g b_{0,i} p_T\left(t - \frac{g}{\beta} - iT\right)}_{s_{ISI}(t)} \\ & + \underbrace{\sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{0}\sum_{g=0}^{\infty} \rho^g b_{k,\ell} e^{j\omega_k(t-(g/\beta))} p_T\left(t - \frac{g}{\beta} - \ell T\right)}_{s_{ACI}(t)}\Bigg] \end{aligned} \tag{9}$$

where $s_B(t)$ is the desired signal (signal of interest), $s_{ISI}(t)$ is the intersymbol interference signal, and $s_{ACI}(t)$ is the adjacent channel interference signal. Then

$$s(t) = r_0(t) = K\left(s_B(t) + s_{ISI}(t) + s_{ACI}(t)\right) \tag{10}$$

We are now interested in writing an expression for $s(t)$ during the detection interval $0 \leq t \leq T$. We do this by sketching $s_B(t)$ from Eq. (9) and then writing another expression which accounts for how the rectangular pulses behave during the detection interval. We then sketch the pulse functions for the first three values of i for $s_{ISI}(t)$ in Eq. (9). We note how they appear during the detection interval, and recognizing a pattern, we write another expression for $s_{ISI}(t)$ for $0 \leq t \leq T$. A similar process is followed to arrive at $s_{ACI}(t)$ for $0 \leq t \leq T$. We now present $s_B(t)$, $s_{ISI}(t)$, and $s_{ACI}(t)$ for $0 \leq t \leq T$ (see Appendix A for details)

$$s_B(t) = \sum_{g=0}^{q-1} \rho^g b_{0,0} p_T\left(\frac{T}{\left(T - \dfrac{g}{\beta}\right)}\left(t - \frac{g}{\beta}\right)\right), \qquad \text{for} \quad 0 \leq t \leq T \tag{11}$$

where $q = T/(1/\beta) = \beta T$ is an integer. Recall that T is the data bit period (s), and β is the single-cavity Fabry-Perot filter's free spectral range (Hz).

$$s_{ISI}(t) = \sum_{i=-L_0}^{-1}\left[\sum_{g=-(1+i)q+1}^{(-iq)-1} \rho^g b_{0,i} p_T\left(\frac{\beta T(t)}{g+(1+i)q}\right)\right.$$

$$\left. + \sum_{g=-iq}^{-(i-1)q-1} \rho^g b_{0,i} p_T\left(\frac{T}{\left(T - \left(\dfrac{g+iq}{\beta}\right)\right)}\left(t - \left(\frac{g+iq}{\beta}\right)\right)\right)\right], \quad \text{for } 0 \leq t \leq T \tag{12}$$

$$
\begin{aligned}
s_{ACI}(t) \; = \; & \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \left(\left[\sum_{g=0}^{q-1} \rho^g b_{k,0} e^{j\omega_k(t-(g/\beta))} p_T \left(\frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right) \right] \right. \\
& + \sum_{\ell=-L}^{-1} \left[\sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \rho^g b_{k,\ell} e^{j\omega_k(t-(g/\beta))} p_T \left(\frac{\beta T(t)}{g+(1+\ell)q} \right) \right. \\
& + \sum_{g=-\ell q}^{-(\ell-1)q-1} \rho^g b_{k,\ell} e^{j\omega_k(t-(g/\beta))} \times \cdots \\
& \left. \left. p_T \left(\frac{T}{\left(T - \left(\frac{g+\ell q}{\beta}\right)\right)} \left(t - \left(\frac{g+\ell q}{\beta}\right)\right) \right) \right] \right), \quad \text{for } 0 \leq t \leq T \quad (13)
\end{aligned}
$$

Recall Eqs. (9) and (10) where

$$s(t) = K(s_B(t) + s_{ISI}(t) + s_{ACI}(t))$$

and

$$K = \sqrt{P}\,(1-\rho)$$

We can see that $s(t)$ in the interval $0 \leq t \leq T$ can now be calculated by substituting Eqs. (11), (12), and (13) into their appropriate positions in Eq. (10).

C. COMPUTATION OF $X = \mathcal{R}\int_0^T |s(t)|^2 dt$, THE SIGNAL COMPONENT OF THE DECISION VARIABLE

Looking at Fig. 1, we see that the filtered optical output of the Fabry-Perot filter $s(t)$ is passed to a photodetector. The output current of the photodetector is $\mathcal{R}\,|s(t)|^2$ where $\mathcal{R}$ is the responsivity of the photodetector (A/W). Thus, we can see that the deterministic signal component of the decision variable at the integrator output is

$$X = \mathcal{R}\int_0^T |s(t)|^2 dt \tag{14}$$

We have already derived $s_B(t)$, $s_{ISI}(t)$, and $s_{ACI}(t)$ for $0 \leq t \leq T$. Directly substituting these three signals into Eq. (10) yields $s(t)$ for $0 \leq t \leq T$. Now our true task is to compute $|s(t)|^2$ and $\int_0^T |s(t)|^2 dt$ as $\mathcal{R}$ is an arbitrary constant.

$$s(t) = \underbrace{\sqrt{P}\,(1-\rho)}_{K}\underbrace{[s_B(t) + s_{ISI}(t) + s_{ACI}(t)]}_{s'(t)} \tag{15}$$

Then

$$|s(t)|^2 = K^2 s'(t) s'(t)^* \tag{16}$$

and dropping the (t) notation for convenience in the three terms of $s'(t)$ and noting that complex conjugation is a linear operator

$$s'(t)^* = s_B^* + s_{ISI}^* + s_{ACI}^* \tag{17}$$

So

$$s'(t)s'(t)^* = (s_B + s_{ISI} + s_{ACI})(s_B^* + s_{ISI}^* + s_{ACI}^*) \tag{18}$$

Multiplication yields

$$\begin{aligned} s'(t)s'(t)^* \;\; = \;\; & s_B s_B^* + s_B s_{ISI}^* + s_B s_{ACI}^* \\ & + s_{ISI} s_B^* + s_{ISI} s_{ISI}^* + s_{ISI} s_{ACI}^* \\ & + s_{ACI} s_B^* + s_{ACI} s_{ISI}^* + s_{ACI} s_{ACI}^* \end{aligned} \tag{19}$$

Rearranging and simplifying yields

$$
\begin{aligned}
|s(t)|^2 = K^2 s'(t)s'(t)^* \quad = \quad K^2 \times \Bigg[& \underbrace{s_B s_B^*}_{|s_B|^2} + \underbrace{s_{ISI} s_{ISI}^*}_{|s_{ISI}|^2} + \underbrace{s_{ACI} s_{ACI}^*}_{|s_{ACI}|^2} \\
& + \underbrace{s_B s_{ISI}^* + s_{ISI} s_B^*} \\
& \qquad\qquad 2Re[s_B s_{ISI}^*] = 2Re[s_{ISI} s_B^*] \\
& + \underbrace{s_B s_{ACI}^* + s_{ACI} s_B^*} \\
& \qquad\qquad 2Re[s_B^* s_{ACI}] = 2Re[s_B s_{ACI}^*] \\
& + \underbrace{s_{ISI} s_{ACI}^* + s_{ACI} s_{ISI}^*} \Bigg] \\
& \qquad\qquad 2Re[s_{ISI}^* s_{ACI}] = 2Re[s_{ISI} s_{ACI}^*]
\end{aligned}
\tag{20}
$$

We will integrate each of these terms over 0 to T to compute $\int_0^T |s(t)|^2 dt$.

Substituting the appropriate definitions into the six terms of Eq. (20) yields 21 terms ("clusters of summations") which are integrated from 0 to T. Also note that we had to create several "gating" functions to turn off integrals when the pulse function products in some of the 21 terms fail to overlap for certain summation indices. Although intuitively obvious, see Appendix A [below Eqs. (104) and (112)] for definitions of the max and min functions which appear in some of the forthcoming equations.

Recall that $K = \sqrt{P}(1-\rho)$ and $K^2 = P(1-\rho)^2$. Then we have

$$
\int_0^T |s(t)|^2 dt = K^2 \int_0^T s'(t)s'(t)^* dt = K^2 \int_0^T |s'(t)|^2 dt \tag{21}
$$

which has the following terms [see Eq. (20)]

$$
K^2 \int_0^T |s_B|^2 dt = P(1-\rho)^2 b_{0,0}^2 \sum_{g=0}^{q-1} \sum_{m=0}^{q-1} \rho^{g+m} \left(T - \left(\frac{\max(g,m)}{\beta} \right) \right) \tag{22}
$$

$$K^2 \int_0^T |s_{ISI}|^2 dt = A_{ISI} + B_{ISI} + C_{ISI} \tag{23}$$

where

$$A_{ISI} = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \varphi_0$$

in which

$$\varphi_0 = \rho^{g+m} b_{0,i} b_{0,r} \left(\frac{\min(g+(1+i)q, m+(1+r)q)}{\beta} \right)$$

$$B_{ISI} = 2P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} G_1(g,i,m,r)$$

in which

$$G_1(g,i,m,r) = \left\{ \begin{array}{ll} \frac{1}{\beta}[(g+(1+i)q)-(m+rq)], & \text{for } m+rq < g+(1+i)q \\ 0, & \text{otherwise} \end{array} \right\}$$

and

$$C_{ISI} = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} \left[T - \left(\frac{\max(g+iq, m+rq)}{\beta} \right) \right]$$

$$K^2 \int_0^T |s_{ACI}|^2 dt = A_{ACI} + B_{ACI} + C_{ACI} + D_{ACI} + E_{ACI} + F_{ACI} \tag{24}$$

where

$$A_{ACI} = P(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n\neq 0}}^{+M/2} \sum_{m=0}^{q-1} \rho^{g+m} \times \varphi_1$$

in which

$$\varphi_1 = \left\{ \begin{array}{ll} b_{k,0} b_{n,0}^* \frac{1}{j(\omega_k - \omega_n)} \times \cdots & \\ \left[e^{j[(\omega_k-\omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} - e^{j[(\omega_k-\omega_n)(1/\beta)\max(g,m) - \omega_k(g/\beta)+\omega_n(m/\beta)]} \right], & \text{for } k \neq n \\ \underbrace{b_{k,0} b_{k,0}^*}_{|b_{k,0}|^2} e^{j[(\omega_k/\beta)(m-g)]} \left[T - \frac{1}{\beta}\max(g,m) \right], & \text{for } k = n \end{array} \right\}$$

$$B_{ACI} = P(1-\rho)^2 \sum_{\substack{k=-M/2\\k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \sum_{\substack{n=-M/2\\n\neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \rho^{g+m} \times \varphi_2$$

in which

$$\varphi_2 = \left\{ \begin{array}{l} b_{k,\ell} b^*_{n,r} \left[\dfrac{e^{j[-\omega_k(g/\beta)+\omega_n(m/\beta)]}}{j(\omega_k-\omega_n)} \left(e^{j[(\omega_k-\omega_n)(1/\beta)\min(g+(1+\ell)q,m+(1+r)q)]} - 1 \right) \right], \\ \hfill \text{for } k \neq n \\ \\ b_{k,\ell} b^*_{k,r} \left(e^{j[(\omega_k/\beta)(m-g)]} \left[\dfrac{1}{\beta} \min(g+(1+\ell)q, m+(1+r)q) \right] \right), \\ \hfill \text{for } k = n \end{array} \right\}$$

$$C_{ACI} = P(1-\rho)^2 \sum_{\substack{k=-M/2\\k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-\ell q}^{-(\ell-1)q-1} \sum_{\substack{n=-M/2\\n\neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} \times \varphi_3$$

in which

$$\varphi_3 = \left\{ \begin{array}{ll} b_{k,\ell} b^*_{n,r} \left[\dfrac{1}{j(\omega_k-\omega_n)} \left[e^{j[(\omega_k-\omega_n)T-\omega_k(g/\beta)+\omega_n(m/\beta)]} \right.\right. & \\ \\ \left.\left. -e^{j[(\omega_k-\omega_n)(1/\beta)\max(g+\ell q,m+rq)-\omega_k(g/\beta)+\omega_n(m/\beta)]} \right] \right], & \text{for } k \neq n \\ \\ b_{k,\ell} b^*_{k,r} e^{j[(\omega_k/\beta)(m-g)]} \left[T - \dfrac{1}{\beta} \max(g+\ell q, m+rq) \right], & \text{for } k = n \end{array} \right\}$$

$$D_{ACI} = P(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n\neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \varphi_4$$

in which

$$\varphi_4 = \left\{ \begin{array}{ll} \rho^{g+m} b_{k,0} b_{n,r}^* \left[\frac{1}{j(\omega_k - \omega_n)} \left[e^{j[(\omega_k-\omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right.\right. & \\ \left.\left. - e^{j[(\omega_k-\omega_n)(1/\beta)\max(g,m+rq) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \right] & \\ + \rho^{g+m} b_{k,0}^* b_{n,r} \left[\frac{1}{[-j(\omega_k - \omega_n)]} \left[e^{-j[(\omega_k-\omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right.\right. & \\ \left.\left. - e^{-j[(\omega_k-\omega_n)(1/\beta)\max(g,m+rq) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \right], & \text{for } k \neq n \\ \left[b_{k,0} b_{k,r}^* e^{j[(\omega_k/\beta)(m-g)]} + b_{k,0}^* b_{k,r} e^{-j[(\omega_k/\beta)(m-g)]} \right] \rho^{g+m} \times \cdots & \\ \left[T - \frac{1}{\beta} \max(g, m+rq) \right], & \text{for } k = n \end{array} \right\}$$

$$E_{ACI} = P(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n\neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \varphi_5$$

in which

$$\varphi_5 = \left\{ \begin{array}{ll} \left[\left(\rho^{g+m} b_{k,0} b^*_{n,r} \frac{1}{j(\omega_k-\omega_n)} \left[e^{j[(\omega_k-\omega_n)((m+(1+r)q)/\beta)-\omega_k(g/\beta)+\omega_n(m/\beta)]}\right.\right.\right. & \\ \left.\left. -e^{j[(\omega_n/\beta)(m-g)]}\right]\right) & \\ +\left(\rho^{g+m} b^*_{k,0} b_{n,r} \frac{1}{[-j(\omega_k-\omega_n)]} \left[e^{-j[(\omega_k-\omega_n)((m+(1+r)q)/\beta)-\omega_k(g/\beta)+\omega_n(m/\beta)]}\right.\right. & \\ \left.\left.\left. -e^{-j[(\omega_n/\beta)(m-g)]}\right]\right)\right] G_2(g,m,r), & \text{for } k \neq n \\ \left[b_{k,0} b^*_{k,r} e^{j[(\omega_k/\beta)(m-g)]} + b^*_{k,0} b_{k,r} e^{-j[(\omega_k/\beta)(m-g)]}\right] \rho^{g+m} \times \cdots & \\ \left(\frac{1}{\beta}[(m+(1+r)q)-g]\right) G_2(g,m,r), & \text{for } k = n \end{array} \right\}$$

where

$$G_2(g,m,r) = \left\{ \begin{array}{ll} 1, & \text{for } g < m+(1+r)q \\ 0, & \text{otherwise} \end{array} \right\}$$

$$F_{ACI} = P(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \sum_{\substack{n=-M/2 \\ n\neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \varphi_6$$

in which

$$\varphi_6 = \left\{ \begin{array}{ll} \Big(\rho^{g+m} b_{k,\ell} b^*_{n,r} \dfrac{1}{j(\omega_k-\omega_n)} \Big[e^{j[(\omega_k-\omega_n)((g+(1+\ell)q)/\beta)-\omega_k(g/\beta)+\omega_n(m/\beta)]} & \\ -e^{j[(\omega_k-\omega_n)((m+rq)/\beta)-\omega_k(g/\beta)+\omega_n(m/\beta)]}\Big] & \\ +\rho^{g+m} b^*_{k,\ell} b_{n,r} \dfrac{1}{[-j(\omega_k-\omega_n)]} \Big[e^{-j[(\omega_k-\omega_n)((g+(1+\ell)q)/\beta)-\omega_k(g/\beta)+\omega_n(m/\beta)]} & \\ -e^{-j[(\omega_k-\omega_n)((m+rq)/\beta)-\omega_k(g/\beta)+\omega_n(m/\beta)]}\Big]\Big) G_3(g,\ell,m,r), & \text{for } k\neq n \\ \Big[b_{k,\ell} b^*_{k,r} e^{j[(\omega_k/\beta)(m-g)]} + b^*_{k,\ell} b_{k,r} e^{-j[(\omega_k/\beta)(m-g)]}\Big] \rho^{g+m} \times \cdots & \\ \left(\dfrac{1}{\beta}[(g+(1+\ell)q)-(m+rq)]\right) G_3(g,\ell,m,r), & \text{for } k=n \end{array} \right\}$$

where

$$G_3(g,\ell,m,r) = \left\{ \begin{array}{ll} 1, & m+rq < g+(1+\ell)q \\ 0, & \text{otherwise} \end{array} \right\}$$

$$K^2 \int_0^T 2Re[s_B s^*_{ISI}]dt = K^2 \int_0^T 2[s_B s_{ISI}]dt =$$

$$2P(1-\rho)^2 \sum_{g=0}^{q-1} \sum_{i=-L_0}^{-1} \sum_{m=-(1+i)q+1}^{(-iq)-1} \rho^{g+m} b_{0,0} b_{0,i} G_4(g,i,m)$$

$$+\,2P(1-\rho)^2 \sum_{g=0}^{q-1} \sum_{i=-L_0}^{-1} \sum_{m=-iq}^{-(i-1)q-1} \rho^{g+m} b_{0,0} b_{0,i} \left[T - \frac{1}{\beta}\max(g, m+iq)\right] \qquad (25)$$

where

$$G_4(g,i,m) = \left\{ \begin{array}{ll} \dfrac{1}{\beta}[(m+(1+i)q)-g], & g < m+(1+i)q \\ 0, & \text{otherwise} \end{array} \right\}$$

$$K^2 \int_0^T 2Re[s_B^* s_{ACI}]dt = K^2 \int_0^T 2Re[s_B s_{ACI}]dt = A_{B\text{-}ACI} + B_{B\text{-}ACI} + C_{B\text{-}ACI} \quad (26)$$

where

$$A_{B\text{-}ACI} = P(1-\rho)^2 \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{m=0}^{q-1} \varphi_7$$

in which

$$\begin{aligned} \varphi_7 &= \rho^{g+m} b_{0,0} b_{k,0} \frac{1}{j\omega_k} \left[e^{j\omega_k (T-(m/\beta))} - e^{j\omega_k ((\max(g,m)-m)/\beta)} \right] \\ &\quad + \rho^{g+m} b_{0,0} b_{k,0}^* \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k (T-(m/\beta))} - e^{-j\omega_k ((\max(g,m)-m)/\beta)} \right] \end{aligned}$$

$$B_{B\text{-}ACI} = P(1-\rho)^2 \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \varphi_8$$

in which

$$\begin{aligned} \varphi_8 &= \rho^{g+m} b_{0,0} b_{k,\ell} \frac{1}{j\omega_k} \left[e^{j\omega_k ((1+\ell)q/\beta)} - e^{j\omega_k ((g-m)/\beta)} \right] G_5(g,\ell,m) \\ &\quad + \rho^{g+m} b_{0,0} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k ((1+\ell)q/\beta)} - e^{-j\omega_k ((g-m)/\beta)} \right] G_5(g,\ell,m) \end{aligned}$$

where

$$G_5(g,\ell,m) = \left\{ \begin{array}{ll} 1, & g < m + (1+\ell)q \\ 0, & \text{otherwise} \end{array} \right\}$$

$$C_{B\text{-}ACI} = P(1-\rho)^2 \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \varphi_9$$

in which

$$\varphi_9 = \rho^{g+m} b_{0,0} b_{k,\ell} \frac{1}{j\omega_k} \left[e^{j\omega_k (T-(m/\beta))} - e^{j\omega_k ((\max(g,m+\ell q)-m)/\beta)} \right]$$
$$+\rho^{g+m} b_{0,0} b^*_{k,\ell} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k (T-(m/\beta))} - e^{-j\omega_k ((\max(g,m+\ell q)-m)/\beta)} \right]$$

$$\begin{aligned} K^2 \int_0^T 2Re[s^*_{ISI} s_{ACI}]dt &= K^2 \int_0^T 2Re[s_{ISI} s_{ACI}]dt \\ &= A_{ISI\text{-}ACI} + B_{ISI\text{-}ACI} + C_{ISI\text{-}ACI} \\ &\quad + D_{ISI\text{-}ACI} + E_{ISI\text{-}ACI} + F_{ISI\text{-}ACI} \end{aligned} \tag{27}$$

where

$$A_{ISI\text{-}ACI} = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{m=0}^{q-1} \varphi_{10}$$

in which

$$\varphi_{10} = \rho^{g+m} b_{0,i} b_{k,0} \frac{1}{j\omega_k} \left[e^{j\omega_k [(1/\beta)(g+(1+i)q)-m/\beta]} - 1 \right] G_6(g,i,m)$$
$$+\rho^{g+m} b_{0,i} b^*_{k,0} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k [(1/\beta)(g+(1+i)q)-m/\beta]} - 1 \right] G_6(g,i,m)$$

where

$$G_6(g,i,m) = \left\{ \begin{array}{ll} 1, & m < g+(1+i)q \\ 0, & \text{otherwise} \end{array} \right\}$$

$$B_{ISI\text{-}ACI} = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \varphi_{11}$$

in which

$$\varphi_{11} = \rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[e^{j\omega_k [(1/\beta)\min(g+(1+i)q, m+(1+\ell)q)-m/\beta]} - e^{j\omega_k [-m/\beta]} \right]$$
$$+\rho^{g+m} b_{0,i} b^*_{k,\ell} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k [(1/\beta)\min(g+(1+i)q, m+(1+\ell)q)-m/\beta]} - e^{-j\omega_k [-m/\beta]} \right]$$

$$C_{ISI\text{-}ACI} = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \varphi_{12}$$

in which

$$\begin{aligned} \varphi_{12} \;=\; & \Big[\rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[e^{j\omega_k[(1/\beta)(g+(1+i)q)-m/\beta]} - e^{j\omega_k[\ell q/\beta]}\right] \\ & + \rho^{g+m} b_{0,i} b^*_{k,\ell} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k[(1/\beta)(g+(1+i)q)-m/\beta]} - e^{-j\omega_k[\ell q/\beta]}\right]\Big] G_7(g,i,\ell,m) \end{aligned}$$

where

$$G_7(g,i,\ell,m) = \left\{ \begin{array}{ll} 1, & m+\ell q < g+(1+i)q \\ 0, & \text{otherwise} \end{array} \right\}$$

$$D_{ISI\text{-}ACI} = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{m=0}^{q-1} \varphi_{13}$$

in which

$$\begin{aligned} \varphi_{13} \;=\; & \rho^{g+m} b_{0,i} b_{k,0} \frac{1}{j\omega_k} \left[e^{j\omega_k[T-(m/\beta)]} - e^{j\omega_k[(1/\beta)\max(g+iq,m)-m/\beta]}\right] \\ & + \rho^{g+m} b_{0,i} b^*_{k,0} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k[T-(m/\beta)]} - e^{-j\omega_k[(1/\beta)\max(g+iq,m)-m/\beta]}\right] \end{aligned}$$

$$E_{ISI\text{-}ACI} = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \varphi_{14}$$

in which

$$\begin{aligned} \varphi_{14} \;=\; & \Big[\rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[e^{j\omega_k[(1/\beta)(1+\ell)q]} - e^{j\omega_k[(1/\beta)(g+iq)-m/\beta]}\right] \\ & + \rho^{g+m} b_{0,i} b^*_{k,\ell} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k[(1/\beta)(1+\ell)q]} - e^{-j\omega_k[(1/\beta)(g+iq)-m/\beta]}\right]\Big] G_8(g,i,\ell,m) \end{aligned}$$

where

$$G_8(g,i,\ell,m) = \left\{ \begin{array}{ll} 1, & g+iq < m+(1+\ell)q \\ & \\ 0, & \text{otherwise} \end{array} \right\}$$

$$F_{ISI\text{-}ACI} = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \varphi_{15}$$

in which

$$\begin{aligned} \varphi_{15} \quad = \quad & \rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[e^{j\omega_k[T-(m/\beta)]} - e^{j\omega_k[(1/\beta)\max(g+iq,m+\ell q)-m/\beta]} \right] \\ & + \rho^{g+m} b_{0,i} b^*_{k,\ell} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k[T-(m/\beta)]} - e^{-j\omega_k[(1/\beta)\max(g+iq,m+\ell q)-m/\beta]} \right] \end{aligned} \quad (28)$$

We have now laid down all terms of

$$\int_0^T |s(t)|^2 dt = K^2 \int_0^T s'(t)s'(t)^* dt = K^2 \int_0^T |s'(t)|^2 dt$$

This will be used to compute the deterministic signal detection statistic X where

$$X = \mathcal{R} \int_0^T |s(t)|^2 dt$$

This will be used to compute probabilities of bit error for the dense WDM system.

D. THE DECISION VARIABLE Y

The decision variable Y appears at the output of the integrator (see Fig. 1). It consists of a signal component X, which was presented in Section C, and a noise component N

$$Y = X + N \quad (29)$$

where

$$X = \mathcal{R} \int_0^T |s(t)|^2 dt \quad (30)$$

and

$$N = \int_0^T n(t)dt \tag{31}$$

We note that $n(t)$ is a zero mean Gaussian random process with two-sided current spectral density N_0 (A^2/Hz), and N is a zero mean Gaussian random variable with variance N_0T. Y is compared to a threshold V_T to determine whether a "0" or "1" bit is presented at the output.

E. PROBABILITY OF BIT ERROR FOR THE DENSE WDM SYSTEM

For a detection threshold V_T and an ACI/ISI bit pattern $\psi_p = \{b_{k,\ell}, b_{0,i}\}$ for $k = -M/2, \cdots, +M/2, k \neq 0$, $\ell = -L, \cdots, 0$, and $i = -L_0, \cdots, -1$; the conditional probability of bit error for the dense WDM system utilizing the decision variable Y described by Eqs. (28)–(30) is given by [2, 4]

$$P_e(\psi_p) = \frac{1}{2}Q\left(\frac{X_1(\psi_p) - V_T}{\sqrt{N_0T}}\right) + \frac{1}{2}Q\left(\frac{V_T - X_0(\psi_p)}{\sqrt{N_0T}}\right) \tag{32}$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-y^2/2}dy \tag{33}$$

X_0 is the value of X given by Eq. (14) or (30) with the value of $\int_0^T |s(t)|^2dt$ obtained by summing Eqs. (22)–(27) when $b_{0,0} = 0$. X_1 is obtained in the same manner with $b_{0,0} = 1$. The average probability of bit error P_b is given by taking the expected value of $P_e(\psi_p)$ given in Eq. (32) over all possible bit patterns ψ_p [2]. Let us define an ACI/ISI bit pattern set $\psi = \{\psi_p\}$; $p = 1, \cdots, NPAT$. $NPAT$ is the total possible number of bit patterns in the set $\psi = \{\psi_p\}$. Then

$$P_b = \underset{\{\psi_p\}}{E}\{P_e(\psi_p)\} \tag{34}$$

If we count up the number of independent bits in ψ_p, there are $M(L+1) + L_0$ bits. Using the assumption that a 0 or 1 is equally probable, we see that there are two possible ways to fill each bit position. Then, by the *generalized multiplication rule*, if we let $NPAT$ be the total number of possible bit patterns in the set $\psi = \{\psi_p\}$

$$NPAT = (2)_1(2)_2 \cdots (2)_{M(L+1)+L_0} = 2^{M(L+1)+L_0} \tag{35}$$

If we assume that all bit patterns in $\psi = \{\psi_p\}$ are equiprobable (an excellent assumption), then by the law of total probability

$$P_b = \frac{1}{2^{M(L+1)+L_0}} P_e(\psi_1) + \frac{1}{2^{M(L+1)+L_0}} P_e(\psi_2) + \cdots + \frac{1}{2^{M(L+1)+L_0}} P_e(\psi_{NPAT})$$

and

$$P_b = \frac{1}{2^{M(L+1)+L_0}} \sum_{p=1}^{NPAT} P_e(\psi_p) \tag{36}$$

III. NUMERICAL RESULTS

We attempt to generate four probability of bit error graphs. We limit our expression for X given by Eq. (14) or (29) and the sum of Eqs. (22)–(27) by setting

$$L_0 = 1 \qquad L = 0 \qquad \phi_k = 0$$

and

$$\omega_k = \frac{2\pi k I}{T}$$

where k is an integer and I is the normalized channel spacing integer ($I > 0$). When we do this, our 21 terms ("clusters of summations") reduce to ten terms ("clusters of summations"). In the interests of brevity, we refer the reader to Appendix B for the details on how we came up with the ten programmable terms of X for this case. Referring to Appendix B, the probability of bit error equations we use to generate these graphs mirror the equations presented in Chapter II, Section E. Referring to Appendix C, one finds the programming strategy for the ten terms and the final graphs, the actual programs for each of the ten terms, and the final four graphing programs which produce one graph each. Each of the four graphs plot probability of bit error versus signal-to-noise ratio $Z = \mathcal{R}\, P\sqrt{T/N_0}$ in dB. Each graph has five traces. One trace shows either Single Channel (SC) operation without Fabry-Perot (FP) filtering or SC operation with FP filtering and without ISI and ACI for comparison with the other four dense WDM traces. The other four traces show probability of bit error versus Z (dB) for four selected values of the normalized channel spacing integer I, or equivalently the number of adjacent channels M. See Appendix C for the relationship between I and M. Each of the four graphs is plotted for a different value βT, the free-spectral range-bit period product. We also note that the four graphs are generated with

$$\rho = \text{Single Cavity Fabry} - \text{Perot Filter Power Reflectivity} = 0.99$$

We now present the values of βT the four values of M, and the corresponding values of I used for each of the four graphs. Each separate value of M, or equivalently I, produces one of the four dense WDM traces for use in comparison with the fifth trace showing single channel operation.

For Fig. 2: $\beta T = 500$ and

$$M = \begin{bmatrix} \underbrace{124}_{I=4} & \underbrace{98}_{I=5} & \underbrace{60}_{I=8} & \underbrace{24}_{I=20} \end{bmatrix}$$

For Fig. 3: $\beta T = 1000$ and

$$M = \begin{bmatrix} \underbrace{198}_{I=5} & \underbrace{164}_{I=6} & \underbrace{110}_{I=9} & \underbrace{48}_{I=20} \end{bmatrix}$$

For Fig. 4: $\beta T = 1500$ and

$$M = \begin{bmatrix} \underbrace{212}_{I=7} & \underbrace{164}_{I=9} & \underbrace{124}_{I=12} & \underbrace{74}_{I=20} \end{bmatrix}$$

For Fig. 5: $\beta T = 2000$ and

$$M = \begin{bmatrix} \underbrace{248}_{I=8} & \underbrace{220}_{I=9} & \underbrace{164}_{I=12} & \underbrace{98}_{I=20} \end{bmatrix}$$

Looking at Figs. 2–5, we see that only Fig. 2 ($\beta T = 500$) is a complete graph. For $\beta T = 500$ we were able to compute all ten terms of X (see Appendix B) for all four given values of M, or equivalently I, and completely compute and simulate all ISI and ACI effects of our complete model given the constraints given at the beginning of this chapter ($L_0 = 1$, $L = 0$, $\phi_k = 0$, $\omega_k = 2\pi kI/T$). For Figs. 3–5 ($\beta T = 1000$, 1500, and 2000, respectively), we were unable to arrive at a solution for Eq. (24) (see Term #5 in Appendix B), which is the very important $K^2 \int_0^T |s_{ACI}|^2 dt$ term. It is very computationally intensive as it involves computing a quadruple summation multiple times

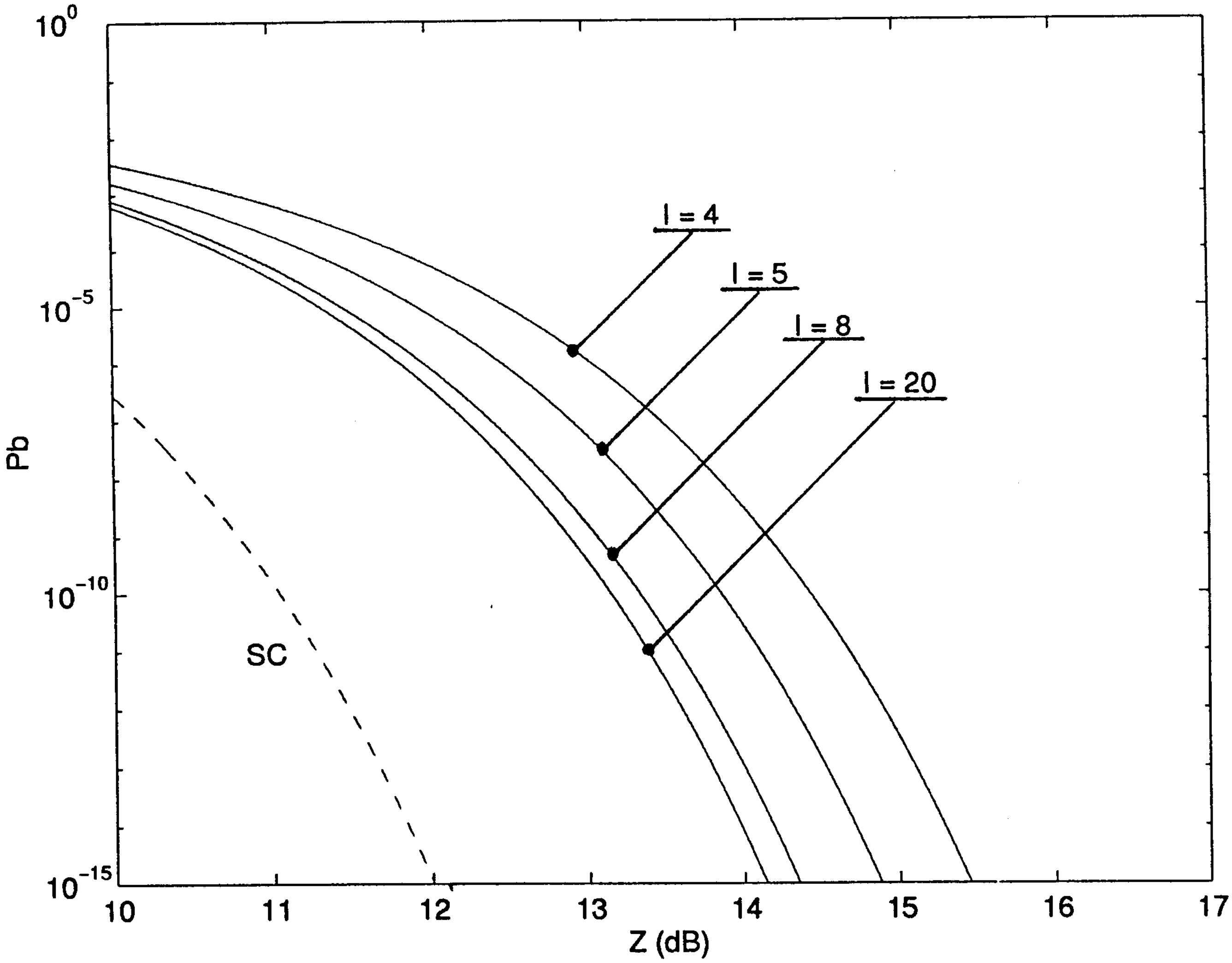


Figure 2: Probability of bit error vs. Z (db) for $\beta T = 500$.

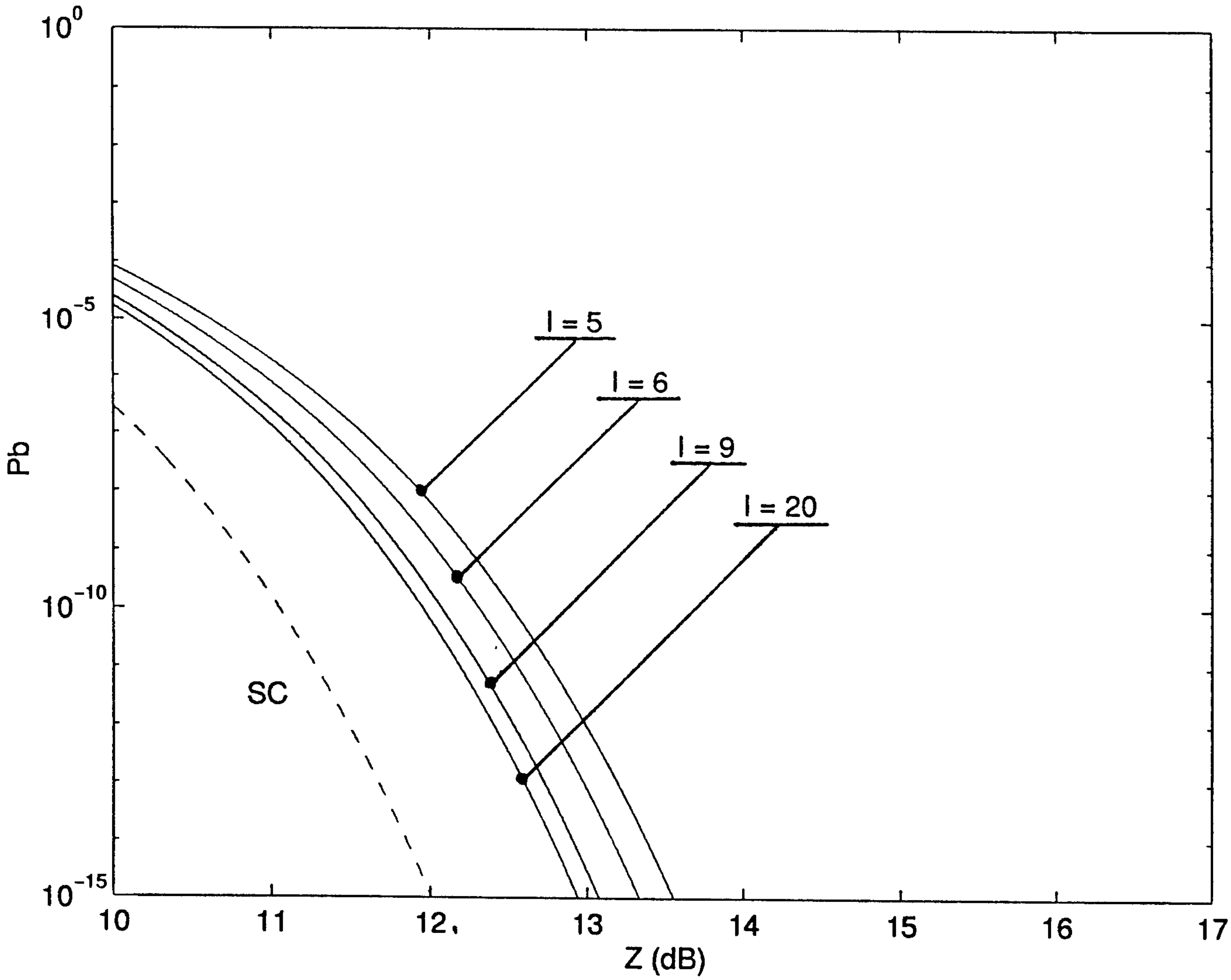


Figure 3: Probability of bit error vs. Z (db) for $\beta T = 1000$. Only $I = 20$ trace accurate (within 1/16 dB).

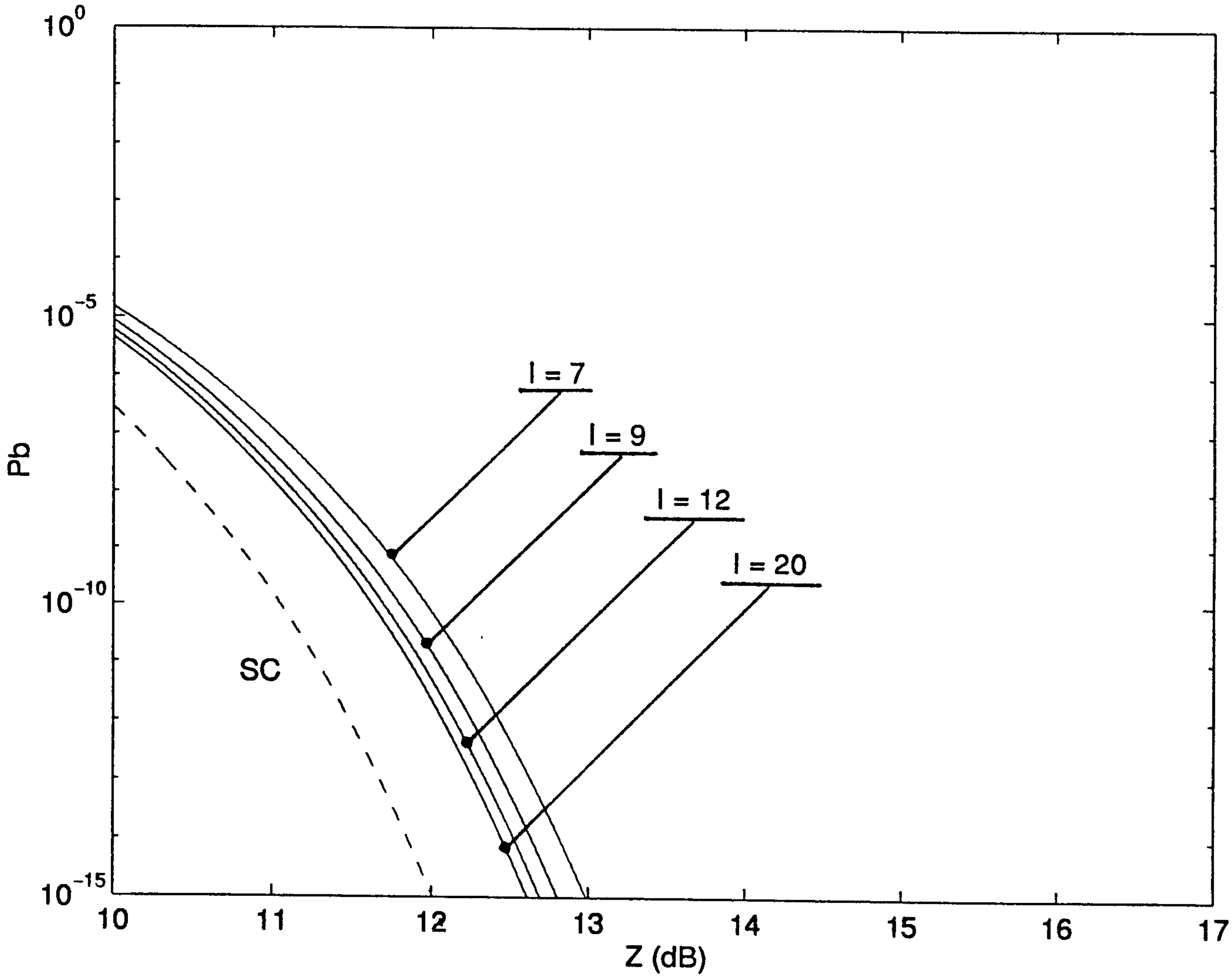


Figure 4: Probability of bit error vs Z (db) for $\beta T = 1500$. Only $I = 20$ trace accurate (within 1/16 dB).

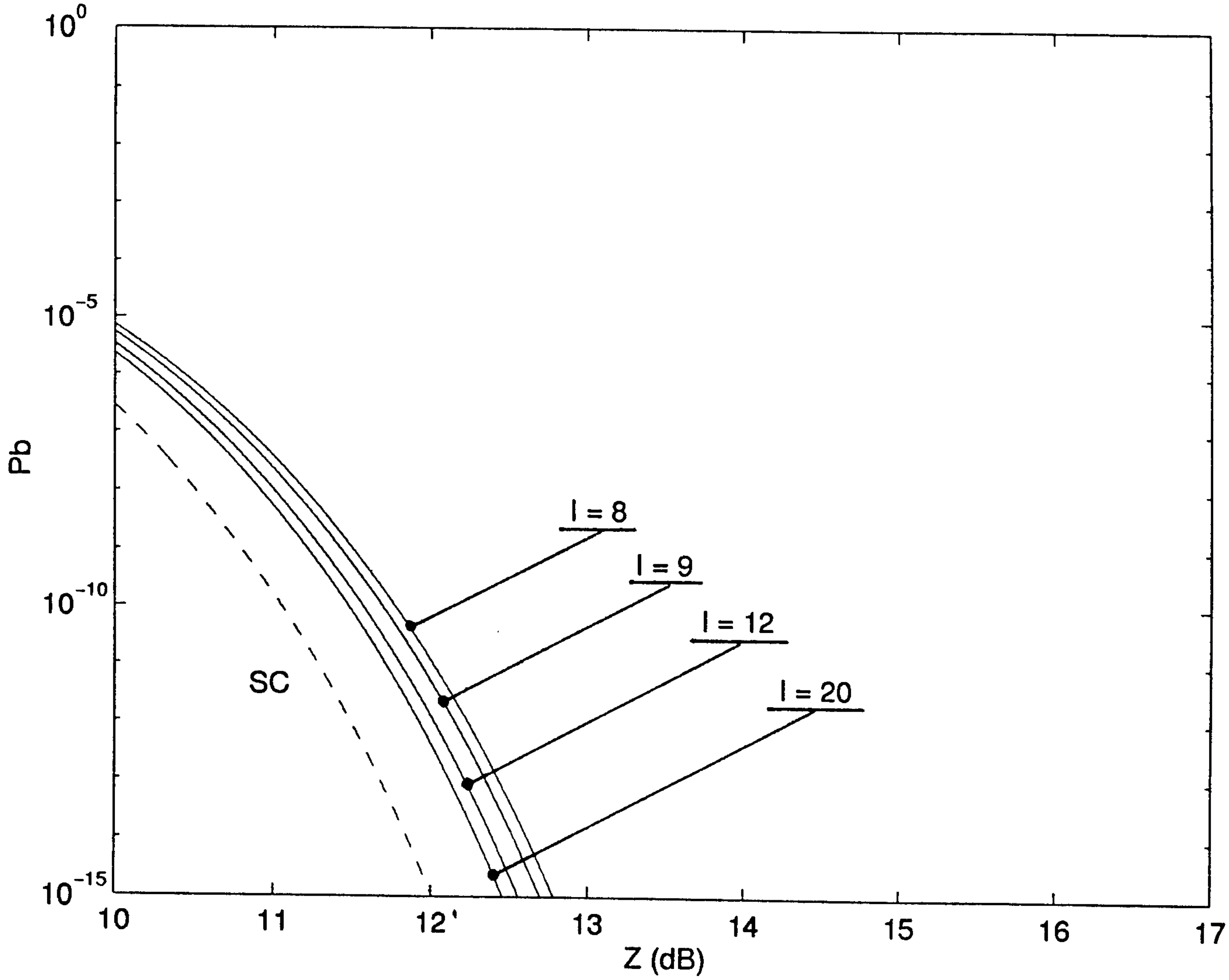


Figure 5: Probability of bit error vs. Z (dB) for $\beta T = 2000$. Only $I = 20$ trace accurate (within 1/5 dB).

for the various adjacent channel bit "loadings" (see Appendix B for the details), as well as having to repeat the same very lengthy calculations for each of the four given values of M, or equivalently I. However, we will still be able to draw some conclusions for the $I = 20$ traces of these graphs as the channel frequency spacing between adjacent channels given by

$$\Delta f = \frac{\omega_k}{2\pi k} = \frac{I}{T} = I R_b \tag{37}$$

where R_b is the channel data rate or channel bit rate. Note that Δf is proportional to I. At these larger frequency spacings, ACI effects become quite negligible. Equation (37) explains why we call I the normalized channel spacing integer as adjacent channels are separated from each other at an integer multiple I of the bit rate R_b. Let us assume a system design of a filter free-spectral-range $\beta = 3800 \times 10^9$ Hz = 3.8 GHz. Then we can say that for $\beta T = 500$ our bit rate $R_b = 7.6$ Gb/sec, for $\beta T = 1000$ our bit rate is $R_b = 3.8$ Gb/sec, for $\beta T = 1500$ our bit rate is $R_b \simeq 2.5$ Gb/sec, and for $\beta T = 2000$ our bit rate is $R_b = 1.9$ Gb/sec. It is noteworthy that, in the time domain model, calculating probability of bit error as presented in Chapter II and Appendix A is easiest for the highest channel data rate R_b, or equivalently the smallest bit period T.

We define power penalty as the increase in signal-to-noise ratio required for the dense WDM system to achieve a 10^{-15} bit error probability over the Single Channel (SC) system which achieves this goal operating at $Z = 12$ dB. We can now make some power penalty statements.

Looking at Fig. 2 ($\beta T = 500$) we see that the power penalty increases as I decreases, or equivalently the number of adjacent channels M increases. Looking at Eq. (37), we see that as I decreases, the interchannel frequency spacing Δf decreases. Thus, it is logical that ACI effects would increase. In short, the channels become much more tightly "packed". Looking at the results for Term #5 in Appendix C when $\beta T = 500$ we see that the magnitude of the contribution for $I = 20$ is only about two to four percent of the contribution for $I = 4$. Returning to Fig. 2 ($\beta T = 500$), we see that the power penalty for $I = 20$ and $M = 24$ is approximately 2 1/8 dB; for $I = 8$ and $M = 60$, the penalty is approximately 2 3/8 dB; for $I = 5$ and $M = 98$, the penalty is approximately 2 7/8 dB; and finally for $I = 4$ and $M = 124$ the penalty is approximately 3 1/2 dB. It is in this case of $I = 4$ and $M = 124$ that we see the true beauty of dense WDM. If we assume $\beta = 3.8$ GHz, then the channel bit rate R_b is 7.6 Gb/sec, and the aggregate system bit rate R_b is 124×7.6 Gb/s = 942 Gb/sec, which is quite close to 1 Tb/sec (1 Tb/sec = 10^{12} b/sec). Thus, for slightly more than doubling the signal-to-noise ratio $Z = \mathcal{R} P \sqrt{T/N_0}$, we have increased the aggregate bit rate of one single fiber close to a terabit per second. Also, the dense WDM system utilizes a noncoherent receiver and, thus, is much cheaper to implement than a coherent optical FDMA network with expensive synchronization circuitry. Looking at Z, however, we see that increasing signal-to-noise ratio is not that easy to do. Upper limits have already been approached in making low-noise current amplifiers; thus, the true technical problem to be solved in the dense WDM system utilizing single-cavity Fabry-Perot filters is to place an optical amplifier before the FP filter (see Fig. 1) to boost the received optical power P. With this problem solved, economical dense WDM systems utilizing single-cavity FP filters with aggregate bit rates with many terabits per second are possible.

Now, in Figs. 3–5 we have already seen that at $I = 20$, the effects of ACI become quite negligible as the channels are spaced farther apart in frequency. Thus, we can conclude that the $I = 20$ probability of bit error traces are accurate for $\beta T = 1000, 1500,$ and 2000. The rest of the traces for $I < 20$ are not. A manuscript in preparation by Tri T. Ha entitled, "A Discrete Time Approach to Sensitivity Analysis of Direct Detection Optical FDMA Networks with OOK Modulation," considers the same problem as this thesis, but uses a discrete time approach. The probability of bit error graphs in this future paper are generated with the same parameters used for this thesis. For the discrete time approach, the $N = \beta T = 1000$ and 1500 traces for $I = 20$ match the $I = 20$ traces for this thesis to within less than 1/16 of a dB, and the $N = \beta T = 2000$ trace for $I = 20$ matches the one in this thesis to within 1/5 of a dB.

Finally, using the case $\beta T = 500$ we feel that our time domain model is very accurate as Fig. 2 agrees with the corresponding discrete time graph for $N = 500$ to within 1/4 of a dB for $I = 4$ and $M = 124$, agrees to within 1/8 of a dB for $I = 5$ and $M = 98$, and matches almost exactly for $I = 8$ and $M = 60$, and $I = 20$ and $M = 24$. These differences are well within the realistic bounds of numerical error as our time domain approach requires many billions of calculations.

IV. CONCLUSION

We have presented a complete model and equation for the probability of bit error of an OOK, dense WDM system employing single-cavity Fabry-Perot filters as channel demultiplexers. Our expression completely models Inter-Symbol Interference (ISI), Adjacent Channel Interference (ACI), and phase offset between Channel 0 and Channel k for all M of the adjacent channels. No simplifying mathematical assumptions have been made to arrive at the final answer contained in Eq. (14), Eq. (29), and the sum of Eqs. (22)–(27). Others [1, 3] have worked on this problem; however, the solution is arrived at in the frequency domain and often involves simplifying mathematical assumptions [1, Eqs. (15), (36), (39) and 3, Eq. (5)]. The closed form expression we have presented for probability of bit error for the dense WDM system, although mathematically rigorous and an extremely complete model, has one significant drawback—it is extremely computationally intensive. The single complete graph for probability of bit error when $\beta T = 500$ took three months to compute using multiple *SPARC 10* workstations working 24 hours per day. Even for our limited case of $L_0 = 1$, $L = 0$, $\omega_k = 2\pi kI/T$, and $\phi_k = 0$, we never arrived at solutions for the other three values of $\beta T = 1000$, 1500, and 2000. The solution is to use a discrete time approach. Again, we are confident of the accuracy of our model as the same problem was attacked independently via discrete time approach, and our graph for $\beta T = 500$ matched the discrete case of $N = \beta T = 500$ to within the bounds of numerical error. Finally, we again mention that to practically implement dense WDM systems with aggregate bit rates of many terabits per second, large increases (doubling, tripling, ...) in the SNR of the dense WDM system over that required for a single channel system are necessary. To accomplish this an optical amplifier will have to be placed in front of the Fabry-Perot filter to greatly increase the filter's received optical power.

APPENDIX A

DERIVATION OF $\int_0^T |s(t)|^2 dt$ FOR USE IN $X = \mathcal{R}\int_0^T |s(t)|^2 dt$

Before we begin the derivation of $\int_0^T |s(t)|^2 dt$, we must first define the variables, equations, and terms involved:

Y — The decision variable appearing at the output of the integrator (see Fig. 1) according to

$$Y = X + N$$

X — The deterministic signal portion of the decision variable

$$X = \mathcal{R}\int_0^T |s(t)|^2 dt$$

$\mathcal{R}|s(t)|^2$ — Output current of the photodetector

$\mathcal{R}$ — Responsivity of the photodetector (A/W)

$n(t)$ — Postdetection thermal noise with two-sided current spectral density N_0 (A^2/Hz)

N — Random signal component of the decision variable Y. N is a zero-mean, Gaussian random variable with variance N_0T

$$N = \int_0^T n(t)dt$$

ρ — Power reflectivity of the single-cavity Fabry-Perot filter

β — Free spectral range of the single-cavity Fabry-Perot filter (Hz)

T — Data bit period (s)

P — Received optical power (W)

$$p_T(t) — \left\{ \begin{array}{ll} 1, & 0 < t < T \\ 0, & \text{otherwise} \end{array} \right\} \tag{38}$$

Desired Channel/Channel of Interest — Channel 0

Adjacent Channel — Channel k, $k = -M/2, \cdots, -1, 1, \cdots, +M/2$ where M is an even integer

$b_{0,0}$ — The detected bit of interest in the interval $0 \leq t \leq T$

$b_{0,i}$ — Bit in Channel 0 during the i^{th} time interval $[iT, (i+1)T]$ where

$$b_{0,i} \in \{0, 1\}$$

$b_{k,\ell}$ — Bit in Channel k during the ℓ^{th} time interval $[\ell T, (\ell+1)T]$ where

$$b_{k,\ell} \in \left\{0, e^{j\phi_k}\right\}$$

and

$$j = \sqrt{-1}$$

ϕ_k — Phase offset between Channel 0 and Channel k. ϕ_k is assumed to be a uniformly distributed random variable between $[0, 2\pi]$

$$\phi_k \sim U[0, 2\pi]$$

ω_k — Radian frequency spacing between Channel 0 and Channel k. Channels are symmetric around Channel 0, i.e.,

$$\omega_k = -\omega_{-k}$$

$h(t)$ — Impulse response of the Channel 0 single cavity Fabry-Perot filter

$$h(t) = (1-\rho) \sum_{g=0}^{\infty} \rho^g \delta\left(t - \frac{g}{\beta}\right) \tag{39}$$

$\delta(t)$ — The *Dirac Delta Function* defined in two parts

$$\delta(t) = \left\{ \begin{array}{ll} 0, & t < 0 \\ 0, & t > 0 \end{array} \right\} \tag{40a}$$

and

$$\int_{0^-}^{0^+} \delta(t)dt = 1 \tag{40b}$$

$b_0(t)$ — Complex baseband equivalent data signal in Channel 0

$$b_0(t) = \sum_{i=-L_0}^{0} b_{0,i} p_T(t - iT) \tag{41}$$

L_0 — An integer greater than zero representing the number of bits in Channel 0 that are trailing the detected bit $b_{0,0}$.

$b_k(t)$ — Complex baseband equivalent data signal in Channel k

$$b_k(t) = \sum_{\ell=-L}^{0} b_{k,\ell} e^{j\omega_k t} p_T(t - \ell T) \tag{42}$$

L — An integer greater than zero representing the number of bits in Channel k which trail the 0^{th} bit in Channel k, $b_{k,0}$.

$r(t)$ — Received complex baseband signal at input of the Channel 0 Fabry-Perot filter

$$r(t) = \sqrt{P}\, b_0(t) + \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sqrt{P}\, b_k(t) \tag{43}$$

We now begin the derivation of $\int_0^T |s(t)|^2 dt$ which allows the computation of

$$X = \mathcal{R} \int_0^T |s(t)|^2 dt$$

where $s(t)$ is the output of the Fabry-Perot filter with an impulse response of $h(t)$

$$\xrightarrow{r(t)} \boxed{h(t)} \xrightarrow{s(t)}$$

$$h(t) = (1-\rho)\sum_{g=0}^{\infty} \rho^g \delta\left(t - \frac{g}{\beta}\right)$$

As stated before

$$r(t) = \sqrt{P}\, b_0(t) + \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sqrt{P}\, b_k(t)$$

Substituting Eqs. (41) and (42) yields

$$r(t) = \sqrt{P}\left(\sum_{i=-L_0}^{0} b_{0,i} p_T(t - iT) + \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{0} b_{k,\ell} e^{j\omega_k t} p_T(t - \ell T) \right) \tag{44}$$

We are interested in the detected bit $b_{0,0}$ during the detection interval $0 \leq t \leq T$, so we need only evaluate $s(t) = r_0(t)$ in the interval $0 \leq t \leq T$

$$s(t) = r_0(t) = r(t) * h(t) = \int_{-\infty}^{\infty} h(t-\tau) r(\tau) d\tau = \int_{-\infty}^{\infty} r(t-\tau) h(\tau) d\tau \tag{45}$$

$$\begin{aligned} s(t) = r_0(t) &= r(t) * h(t) \\ &= h(t) * r(t) \\ &= h(t) * \left(\sqrt{P}\, b_0(t) + \sqrt{P} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} b_k(t) \right) \end{aligned} \tag{46}$$

As convolution is a linear operator and distributes over addition

$$s(t) = r_0(t) = \left(\sqrt{P}\,(h(t) * b_0(t)) + \sqrt{P} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} h(t) * b_k(t) \right) \tag{47}$$

$$s(t) = r_0(t) = \sqrt{P}\left(h(t) * b_0(t) \; + \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} h(t) * b_k(t) \right) \tag{48}$$

Now looking at the term within the parentheses

$$h(t) * b_0(t) \; + \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} h(t) * b_k(t) = h(t) * \sum_{i=-L_0}^{0} b_{0,i} p_T(t - iT)$$

$$+ \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} h(t) * \sum_{\ell=-L}^{0} b_{k,\ell} e^{j\omega_k t} p_T(t - \ell T) \tag{49}$$

Applying the distributive property of convolution

$$h(t) * b_0(t) + \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} h(t) * b_k(t) = \sum_{i=-L_0}^{0} h(t) * (b_{0,i} p_T(t - iT))$$

$$+ \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{0} h(t) * \left(b_{k,\ell} e^{j\omega_k t} p_T(t - \ell T) \right) \tag{50}$$

Expressing convolution in terms of its integral definition

$$h(t) * b_0(t) + \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} h(t) * b_k(t) = \underbrace{\sum_{i=-L_0}^{0} \int_{-\infty}^{\infty} h(t - \tau) \left[b_{0,i} p_T(\tau - iT) \right] d\tau}_{A_1}$$

$$\underbrace{+ \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{0} \int_{-\infty}^{\infty} h(t - \tau) \left[b_{k,\ell} e^{j\omega_k \tau} p_T(\tau - \ell T) \right] d\tau}_{B_1} \tag{51}$$

Substituting the expression for $h(t - \tau)$ [Eq. (39)] into term A_1, above

$$A_1 = \sum_{i=-L_0}^{0} \int_{-\infty}^{\infty} (1 - \rho) \sum_{g=0}^{\infty} \rho^g \delta \left(t - \tau - \frac{g}{\beta} \right) b_{0,i} p_T(\tau - iT) d\tau \tag{52}$$

Factoring $(1 - \rho)$ and interchanging the order of integration and summation yields

$$A_1 = (1 - \rho) \sum_{i=-L_0}^{0} \sum_{g=0}^{\infty} \rho^g b_{0,i} \int_{-\infty}^{\infty} \delta \left(t - \tau - \frac{g}{\beta} \right) p_T(\tau - iT) d\tau \tag{53}$$

Using the identity: $\int_{-\infty}^{+\infty} x(t)\delta(t-t_0)dt = x(t_0)$, the integral evaluates to $p_T(t - g/\beta - iT)$. We then have the following for A_1

$$\begin{aligned} A_1 &= \sum_{i=-L_0}^{0} \int_{-\infty}^{\infty} h(t-\tau)\left[b_{0,i}p_T(\tau - iT)\right] d\tau \\ &= (1-\rho) \sum_{i=-L_0}^{0} \sum_{g=0}^{\infty} \rho^g b_{0,i} p_T\left(t - \frac{g}{\beta} - iT\right) \end{aligned} \tag{54}$$

Recalling the expression for

$$B_1 = \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{0} \int_{-\infty}^{\infty} h(t-\tau)\left[b_{k,\ell}e^{j\omega_k\tau}p_T(\tau - \ell T)\right] d\tau \tag{55}$$

Substituting the expression for $h(t-\tau)$ given by Eq. (39) into Eq. (55) above, we obtain

$$B_1 = \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{0} \int_{-\infty}^{\infty} (1-\rho) \sum_{g=0}^{\infty} \rho^g \delta\left(t - \tau - \frac{g}{\beta}\right)\left[b_{k,\ell}e^{j\omega_k\tau}p_T(\tau - \ell T)\right] d\tau \tag{56}$$

Factoring $1-\rho$, interchanging the order of integration and summation, and factoring $b_{k,\ell}$ and ρ^g from the integral

$$B_1 = (1-\rho) \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{0} \sum_{g=0}^{\infty} \rho^g b_{k,\ell} \int_{-\infty}^{\infty} \delta\left(t - \tau - \frac{g}{\beta}\right) e^{j\omega_k\tau}p_T(\tau - \ell T) d\tau \tag{57}$$

Applying the property $\int_{-\infty}^{+\infty} x(t)\delta(t-t_0)dt = x(t_0)$

$$\int_{-\infty}^{\infty} \delta\left(t - \tau - \frac{g}{\beta}\right) e^{j\omega_k\tau}p_T(\tau - \ell T) d\tau = e^{j\omega_k(t-(g/\beta))} p_T\left(t - \frac{g}{\beta} - \ell T\right) \tag{58}$$

Substituting Eq. (58) into Eq. (57) yields

$$\begin{aligned} B_1 &= \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{0} \int_{-\infty}^{\infty} h(t-\tau)\left[b_{k,\ell}e^{j\omega_k\tau}p_T(\tau - \ell T)\right] d\tau \\ &= (1-\rho) \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{0} \sum_{g=0}^{\infty} \rho^g b_{k,\ell} e^{j\omega_k(t-(g/\beta))} p_T\left(t - \frac{g}{\beta} - \ell T\right) \end{aligned} \tag{59}$$

Substituting Eq. (54) and Eq. (59) into Eq. (51) and then substituting this result into Eq. (48) yields

$$
\begin{aligned}
s(t) &= r_0(t) = \sqrt{P}\Bigg[\underbrace{(1-\rho)\sum_{i=-L_0}^{0}\sum_{g=0}^{\infty}\rho^g b_{0,i}p_T\left(t-\frac{g}{\beta}-iT\right)}_{A_1} \\
&\quad \underbrace{+(1-\rho)\sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2}\ \sum_{\ell=-L}^{0}\sum_{g=0}^{\infty}\rho^g b_{k,\ell}e^{j\omega_k(t-(g/\beta))}p_T\left(t-\frac{g}{\beta}-\ell T\right)\Bigg]}_{B_1} \qquad (60)
\end{aligned}
$$

We can now factor out $(1-\rho)$ from our expression above. Then

$$
\begin{aligned}
s(t) &= r_0(t) = \sqrt{P}\,(1-\rho)\Bigg[\sum_{i=-L_0}^{0}\sum_{g=0}^{\infty}\rho^g b_{0,i}p_T\left(t-\frac{g}{\beta}-iT\right) \\
&\quad +\sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2}\ \sum_{\ell=-L}^{0}\sum_{g=0}^{\infty}\rho^g b_{k,\ell}e^{j\omega_k(t-(g/\beta))}p_T\left(t-\frac{g}{\beta}-\ell T\right)\Bigg] \qquad (61)
\end{aligned}
$$

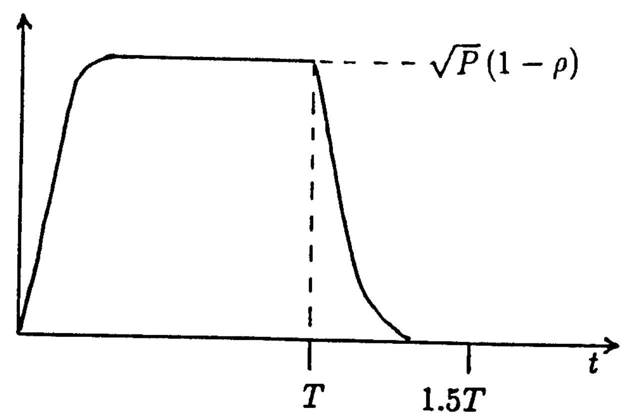
Looking at Eq. (61). We separate the terms involving the detected bit of interest in Channel 0, $b_{0,0}$, the trailing bits in Channel 0, $b_{0,i}$ $i \neq 0$, and the terms involving bits in the other channels, $b_{k,\ell}$, $k \neq 0$

$$
\begin{aligned}
s(t) &= r_0(t) = \underbrace{\sqrt{P}\,(1-\rho)}_{K}\Bigg[\underbrace{\sum_{g=0}^{\infty}\rho^g b_{0,0}p_T\left(t-\frac{g}{\beta}\right)}_{s_B(t)} \\
&\quad +\underbrace{\sum_{i=-L_0}^{-1}\sum_{g=0}^{\infty}\rho^g b_{0,i}p_T\left(t-\frac{g}{\beta}-iT\right)}_{s_{ISI}(t)} \\
&\quad +\underbrace{\sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2}\ \sum_{\ell=-L}^{0}\sum_{g=0}^{\infty}\rho^g b_{k,\ell}e^{j\omega_k(t-(g/\beta))}p_T\left(t-\frac{g}{\beta}-\ell T\right)\Bigg]}_{s_{ACI}(t)} \qquad (62)
\end{aligned}
$$

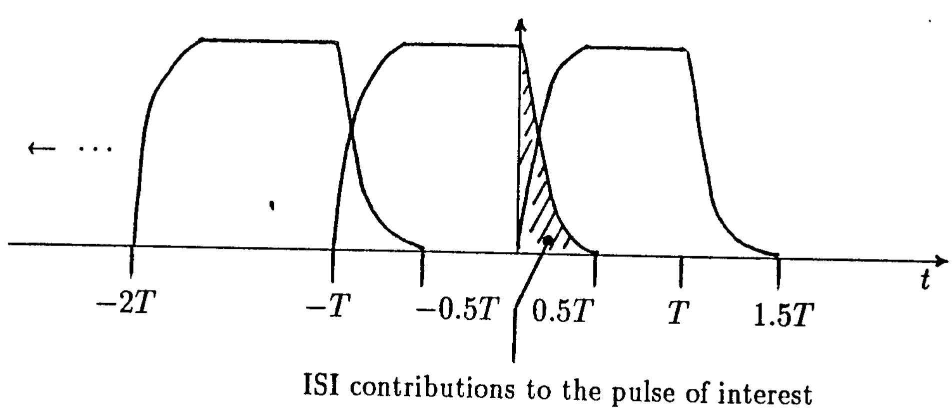
where $s_B(t)$ is the desired signal, $s_{ISI}(t)$ is the Intersymbol Interference (ISI) signal, and $s_{ACI}(t)$ is the Adjacent Channel Interference (ACI) signal. Using this compact notation, we have

$$s(t) = r_0(t) = K\left(s_B(t) + s_{ISI}(t) + s_{ACI}(t)\right) \tag{63}$$

Sketching $s_B(t)$ for $b_{0,0} = 1$



We can also sketch the intersymbol interference due to the trailing pulses in Channel 0



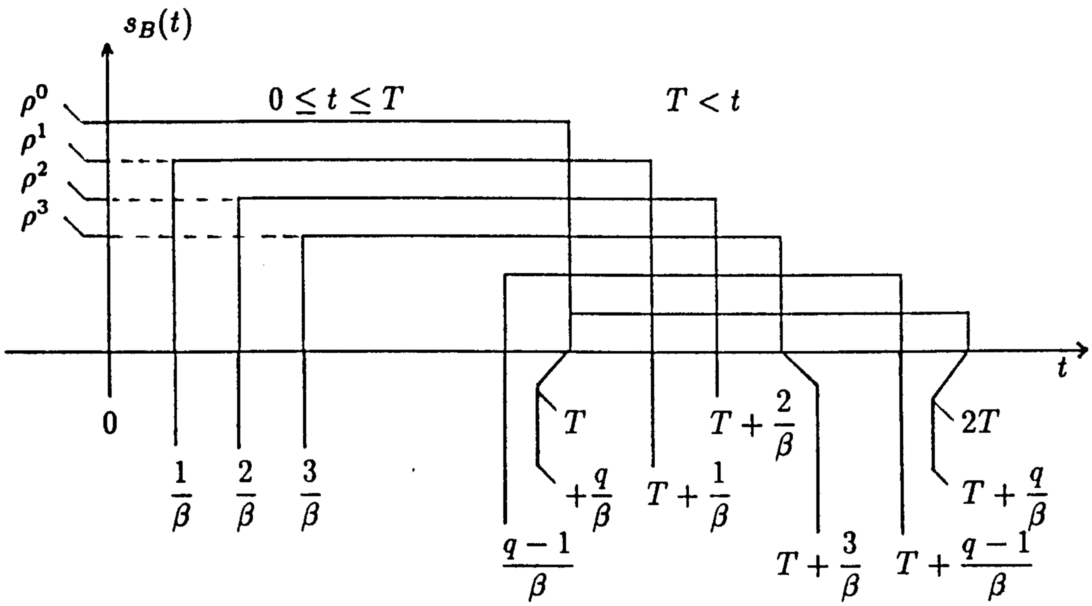
Now let us assume in one pulse interval T that

$$\frac{T}{\frac{1}{\beta}} = q \tag{64}$$

where $1/\beta$ divides evenly into the period T making q an integer quantity. We are only interested in the detection interval $0 \leq t \leq T$, so we only need to evaluate $s(t) = r_0(t)$ in the interval $0 \leq t \leq T$. Looking at the desired signal, which is the system response to the bit of interest $b_{0,0}$

$$s_B(t) = \sum_{g=0}^{\infty} \rho^g b_{0,0} p_T \left(t - \frac{g}{\beta}\right), \qquad \text{for } 0 \leq t < \infty \tag{65}$$

Assuming $b_{0,0} = 1$, $s_B(t)$ appears as



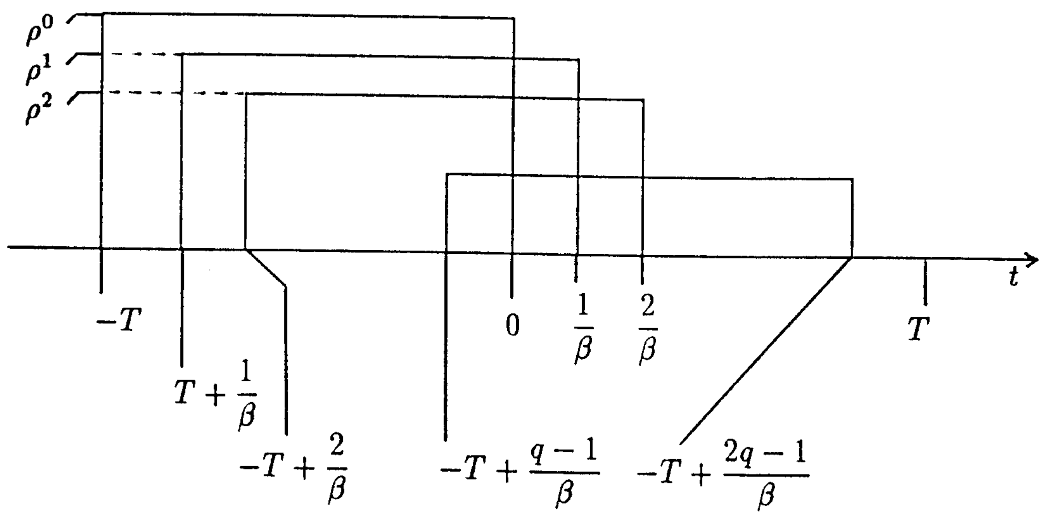
We can see that, during the interval $0 \leq t \leq T$, the response is a sum of scaled, shifted rectangular pulses. Therefore

$$s_B(t) = \sum_{g=0}^{q-1} \rho^g b_{0,0} p_T \left(\frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right), \qquad \text{for } 0 \leq t \leq T \tag{66}$$

We now direct out efforts towards developing an expression for $s_{ISI}(t)$ during the interval $0 \leq t \leq T$. We begin by examining the ISI term for $i = -1$ in Eq. (62), denoted by $-1ISI$.

$$-1\,ISI = \sum_{g=0}^{\infty} \rho^g b_{0,-1} p_T \left(t - \frac{g}{\beta} + 1T\right), \qquad \text{for } -T \leq t < \infty \tag{67}$$

By examing the sketch of $-1ISI$



we see that $-1ISI$ will also be a sum of shifted, scaled rectangular pulses for $0 \leq t \leq T$. The $-1ISI$ expression will have two parts, the first of which is

$$-1\,ISI(1) = \sum_{g=1}^{q-1} \rho^g b_{0,-1} p_T \left(\frac{\beta T(t)}{g} \right), \qquad \text{for } 0 \leq t \leq T \tag{68}$$

But with the q^{th} shift we no longer sum scaled pulses beginning at $t = 0$ and ending at $1/\beta, 2/\beta, \cdots$, but now we sum pulses which progressively scale and begin at $1/\beta, 2/\beta, \cdots$, and end at $t = T$. This second part of the expression looks much like the expression for $s_B(t)$ during the interval $0 \leq t \leq T$ [Eq. (66)]. Thus, we may write the second part of $-1ISI$

$$-1\,ISI(2) = \sum_{g=q}^{2q-1} \rho^g b_{0,-1} p_T \left(\frac{T}{\left(T - \left(\frac{g-q}{\beta}\right)\right)} \left(t - \left(\frac{g-q}{\beta}\right)\right) \right), \qquad \text{for } 0 \leq t \leq T \tag{69}$$

Then, since $-1\,ISI = -1\,ISI(1) + \ -1\,ISI(2)$ for $0 \leq t \leq T$ we have

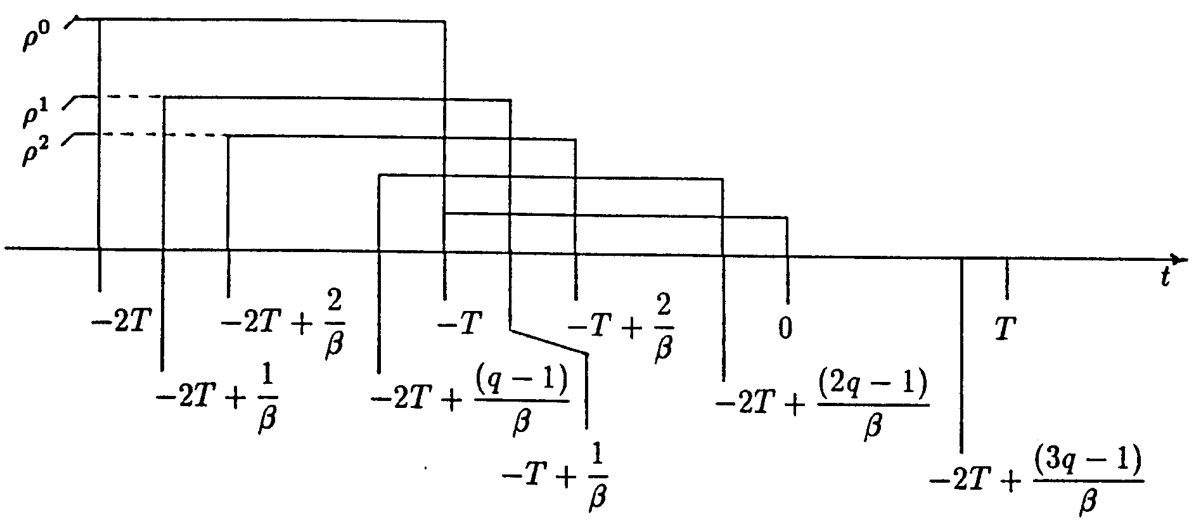
$$-1\,ISI = \sum_{g=1}^{q-1} \rho^g b_{0,-1} p_T \left(\frac{\beta T(t)}{g} \right)$$

$$+\sum_{g=q}^{2q-1} \rho^g b_{0,-1} p_T \left(\frac{T}{\left(T - \left(\frac{g-q}{\beta}\right)\right)} \left(t - \left(\frac{g-q}{\beta}\right)\right) \right), \qquad \text{for } 0 \le t \le T \qquad (70)$$

Examine the ISI term for $i = -2$ in Eq. (62), $-2ISI$

$$-2\,ISI = \sum_{g=0}^{\infty} b_{0,-2} \rho^g p_T \left(t - \frac{g}{\beta} + 2T\right), \qquad \text{for } -2T \le t < \infty \qquad (71)$$

We sketch $-2ISI$



By analyzing this sketch, we can obtain

$$-2\,ISI(1) = \sum_{g=q+1}^{2q-1} \rho^g b_{0,-2} p_T \left(\frac{\beta T(t)}{(g-q)}\right), \qquad \text{for } 0 \le t \le T \qquad (72)$$

and

$$-2\,ISI(2) = \sum_{g=2q}^{3q-1} \rho^g b_{0,-2} p_T \left(\frac{T}{\left(T - \left(\frac{g-2q}{\beta}\right)\right)} \left(t - \left(\frac{g-2q}{\beta}\right)\right) \right),$$

$$\text{for } 0 \le t \le T \qquad (73)$$

Note: These rectangular pulses shift into the interval $0 \le t \le T$ on the $2q^{\text{th}}$ shift.

Similar to $-1ISI$: $\quad -2\,ISI = -2\,ISI(1) + \; -2\,ISI(2)$ for the interval $0 \leq t \leq T$

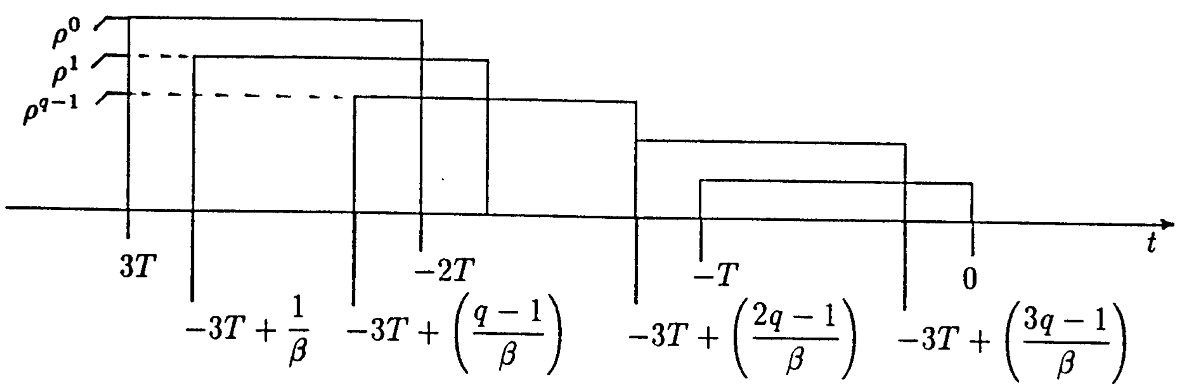
$$-2\,ISI = \sum_{g=q+1}^{2q-1} \rho^g b_{0,-2} p_T \left(\frac{\beta T(t)}{g-q} \right)$$

$$+ \sum_{g=2q}^{3q-1} \rho^g b_{0,-2} p_T \left(\frac{T}{\left(T - \left(\frac{g-2q}{\beta} \right) \right)} \left(t - \left(\frac{g-2q}{\beta} \right) \right) \right), \qquad \text{for } 0 \leq t \leq T \qquad (74)$$

We now examine the $-3ISI$ term to ensure recognition of a pattern, if any, that will allow us to write an expression for $s_{ISI}(t)$ for $0 \leq t \leq T$.

$$-\,3\,ISI = \sum_{g=0}^{\infty} \rho^g b_{0,-3} p_T \left(t - \frac{g}{\beta} + 3T \right), \qquad \text{for } -3T \leq t < \infty \qquad (75)$$

From the sketch of $-3ISI$



we may write directly

$$-3\,ISI = \sum_{g=2q+1}^{3q-1} \rho^g b_{0,-3} p_T \left(\frac{\beta T(t)}{g-2q} \right)$$

$$+ \sum_{g=3q}^{4q-1} \rho^g b_{0,-3} p_T \left(\frac{T}{\left(T - \left(\frac{g-3q}{\beta} \right) \right)} \left(t - \left(\frac{g-3q}{\beta} \right) \right) \right), \qquad \text{for } 0 \leq t \leq T \qquad (76)$$

Then from Eqs. (70), (74), (76), the expressions for the $-1ISI$, $-2ISI$, and $-3ISI$ pulses, we are ready to use the pattern developed to write an expression for the ISI between $0 \leq t \leq T$. Recalling

$$s_{ISI}(t) = \sum_{i=-L_0}^{-1} \sum_{g=0}^{\infty} \rho^g b_{0,i} p_T \left(t - \frac{g}{\beta} - iT \right), \qquad \text{for } -L_0 T \leq t < \infty \tag{77}$$

we may write directly from the pattern developed

$$s_{ISI}(t) = \sum_{i=-L_0}^{-1} \left[\sum_{g=-(1+i)q+1}^{(-iq)-1} \rho^g b_{0,i} p_T \left(\frac{\beta T(t)}{g + (1+i)q} \right) \right.$$

$$\left. + \sum_{g=-iq}^{-(i-1)q-1} \rho^g b_{0,i} p_T \left(\frac{T}{\left(T - \left(\frac{g+iq}{\beta} \right) \right)} \left(t - \left(\frac{g+iq}{\beta} \right) \right) \right) \right], \qquad \text{for } 0 \leq t \leq T \tag{78}$$

We now have expressions for $s_B(t)$ and $s_{ISI}(t)$, during the interval $0 \leq t \leq T$. We have only the ACI term $s_{ACI}(t)$ to calculate

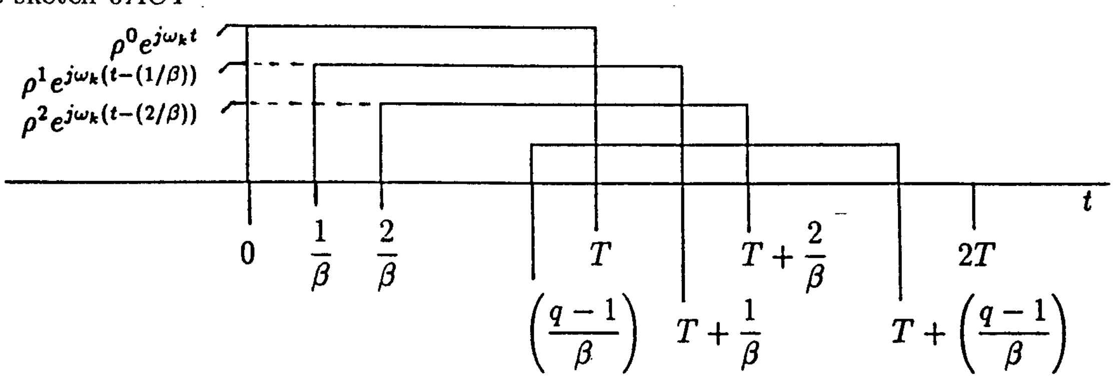
$$s_{ACI}(t) = \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \underbrace{\sum_{\ell=-L}^{0} \sum_{g=0}^{\infty} \rho^g b_{k,\ell} e^{j\omega_k (t-(g/\beta))} p_T \left(t - \frac{g}{\beta} - \ell T \right)}_{*} \quad \text{for } -LT \leq t < \infty \tag{79}$$

* *Note:* This portion of $s_{ACI}(t)$ is almost exactly like the ISI term with which we just worked [see Eq. (62)]. The only difference is the complex exponential factor.

Looking at the 0^{th} term of the ACI in the k^{th} channel

$$0ACI = \sum_{g=0}^{\infty} \rho^g b_{k,0} e^{j\omega_k (t-(g/\beta))} p_T \left(t - \frac{g}{\beta} \right), \qquad \text{for } 0 \leq t < \infty \tag{80}$$

we sketch $0ACI$



We see, that in a manner similar to the $s_B(t)$ signal, the pulses add as before, but now the amplitude of the pulses is modulated by $\rho^g e^{j\omega_k(t-(g/\beta))}$ versus just ρ^g before. Then

$$0ACI = \sum_{g=0}^{q-1} \rho^g b_{k,0} e^{j\omega_k(t-(g/\beta))} p_T \left(\frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right), \qquad \text{for } 0 \leq t \leq T \qquad (81)$$

The $-1ACI$ will be similar to the $-1ISI$ term [see Eq. (70)]. However, now we not only have ρ^g but also the complex exponential gated by the pulses in the summation.

$$-1\ ACI = \sum_{g=1}^{q-1} \rho^g b_{k,-1} e^{j\omega_k(t-(g/\beta))} p_T \left(\frac{\beta T(t)}{g} \right)$$
$$+ \sum_{g=q}^{2q-1} \rho^g b_{k,-1} e^{j\omega_k(t-(g/\beta))} p_T \left(\frac{T}{\left(T - \left(\frac{g-q}{\beta}\right)\right)} \left(t - \left(\frac{g-q}{\beta}\right)\right) \right), \quad \text{for } 0 \leq t \leq T \qquad (82)$$

The derivation of $-2ACI$ and $-3ACI$ will proceed in a similar manner as $-2ISI$ and $-3ISI$ [Eqs. (74) and (76)]. We see that two different type pulses gate the product of ρ^g and a complex exponential term. These gated terms are then summed over the period 0 to T. We then use our expressions for $0ACI$, $-1ACI$, ... to write an expression for $s_{ACI}(t)$ during the interval $0 \leq t \leq T$ in a manner similar to the $s_{ISI}(t)$ expression. We may now write the final expression for the ACI

$$\begin{aligned} s_{ACI}(t) \quad = \quad & \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \left(\left[\sum_{g=0}^{q-1} \rho^g b_{k,0} e^{j\omega_k(t-(g/\beta))} p_T \left(\frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right) \right] \right. \\ & + \sum_{\ell=-L}^{-1} \left[\sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \rho^g b_{k,\ell} e^{j\omega_k(t-(g/\beta))} p_T \left(\frac{\beta T(t)}{g + (1+\ell)q} \right) \right. \\ & + \sum_{g=-\ell q}^{-(\ell-1)q-1} \rho^g b_{k,\ell} e^{j\omega_k(t-(g/\beta))} \times \cdots \end{aligned}$$

$$p_T\left(\frac{T}{\left(T-\left(\frac{g+\ell q}{\beta}\right)\right)}\left(t-\left(\frac{g+\ell q}{\beta}\right)\right)\right)\Big]\Big]\Bigg), \qquad \text{for } 0\le t\le T \tag{83}$$

We have now derived $s_B(t)$, $s_{ISI}(t)$, and $s_{ACI}(t)$ during the interval $0 \le t \le T$. We now develop the expressions needed to compute $|s(t)|^2$ and $\int_0^T |s(t)|^2 dt$. Substituting the expressions for $s_B(t)$, $s_{ISI}(t)$, and $s_{ACI}(t)$ for the interval $0 \le t \le T$ into Eq. (63) yields $s(t)$ in the interval $0 \le t \le T$

$$s(t) = \underbrace{\sqrt{P}\,(1-\rho)}_{K}\underbrace{[s_B(t)+s_{ISI}(t)+s_{ACI}(t)]}_{s'(t)}, \qquad \text{for } 0\le t\le T \tag{84}$$

where $s_B(t)$, $s_{ISI}(t)$, and $s_{ACI}(t)$ are described by Eqs. (66), (78), and (83) respectively. Then

$$|s(t)|^2 = K^2 s'(t)s'(t)^* \tag{85}$$

Dropping the (t) notation for convenience in the three terms of $s'(t)$ and noting that complex conjugation is a linear operator

$$s'(t)^* = s_B^* + s_{ISI}^* + s_{ACI}^* \tag{86}$$

So

$$s'(t)s'(t)^* = (s_B + s_{ISI} + s_{ACI})(s_B^* + s_{ISI}^* + s_{ACI}^*) \tag{87}$$

Multiplication yields

$$\begin{aligned} s'(t)s'(t)^* \;=\; & s_B s_B^* + s_B s_{ISI}^* + s_B s_{ACI}^* \\ & + s_{ISI} s_B^* + s_{ISI} s_{ISI}^* + s_{ISI} s_{ACI}^* \\ & + s_{ACI} s_B^* + s_{ACI} s_{ISI}^* + s_{ACI} s_{ACI}^* \end{aligned} \tag{88}$$

Rearranging and simplifying yields

$$|s(t)|^2 = K^2 s'(t)s'(t)^* \;=\; K^2 \times \Bigg[\underbrace{s_B s_B^*}_{|s_B|^2} + \underbrace{s_{ISI} s_{ISI}^*}_{|s_{ISI}|^2} + \underbrace{s_{ACI} s_{ACI}^*}_{|s_{ACI}|^2}$$

$$+\underbrace{s_B s_{ISI}^* + s_{ISI} s_B^*}_{2Re[s_B s_{ISI}^*] = 2Re[s_{ISI} s_B^*]}$$

$$+\underbrace{s_B s_{ACI}^* + s_{ACI} s_B^*}_{2Re[s_B^* s_{ACI}] = 2Re[s_B s_{ACI}^*]}$$

$$+\underbrace{s_{ISI} s_{ACI}^* + s_{ACI} s_{ISI}^*}_{2Re[s_{ISI}^* s_{ACI}] = 2Re[s_{ISI} s_{ACI}^*]}\Bigg] \tag{89}$$

We will integrate each of these terms over 0 to T to compute $\int_0^T |s(t)|^2 dt$. To avoid confusion, we must realize that constant K^2 must be carried through to the final calculation of all quantities. Recalling Eq. (66)

$$s_B = \sum_{g=0}^{q-1} \rho^g b_{0,0} p_T \left(\frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right) \tag{90}$$

We know $b_{0,i} \in (0,1)$, ρ^g is real, and $p_T(\cdot)$ is also real. Then

$$s_B = s_B^* \tag{91}$$

and

$$|s_B|^2 = \left(\sum_{g=0}^{q-1} \rho^g b_{0,0} p_T \left(\frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right) \right)^2 \tag{92}$$

Factoring $b_{0,0}$ as it is a constant to the summation, we obtain

$$|s_B|^2 = b_{0,0}^2 \left(\sum_{g=0}^{q-1} \rho^g p_T \left(\frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right) \right)^2 \tag{93}$$

Performing the squaring operation yields

$$|s_B|^2 = b_{0,0}^2 \left(\sum_{g=0}^{q-1} \sum_{m=0}^{q-1} \rho^{g+m} p_T \left(\frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right) p_T \left(\frac{T}{\left(T - \frac{m}{\beta}\right)} \left(t - \frac{m}{\beta}\right) \right) \right) \tag{94}$$

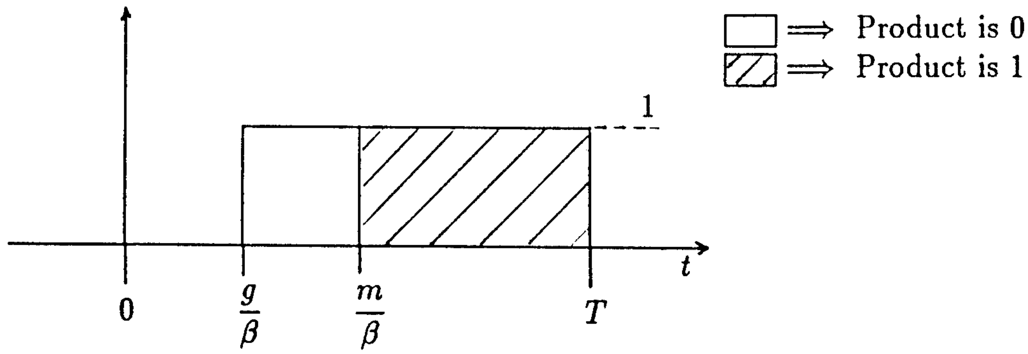
Then

$$\begin{aligned} K^2|s_B|^2 \quad &= \quad P(1-\rho)^2 b_{0,0}^2 \left[\sum_{g=0}^{q-1} \sum_{m=0}^{q-1} \rho^{g+m} p_T \left(\frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right) \times \cdots \right. \\ &\left. p_T \left(\frac{T}{\left(T - \frac{m}{\beta}\right)} \left(t - \frac{m}{\beta}\right) \right) \right], \qquad \text{for } 0 \le t \le T \end{aligned} \tag{95}$$

Now we want to compute $\int_0^T K^2|s_B|^2 dt$ [see Eq. (89)].

$$\begin{aligned} \int_0^T K^2|s_B|^2 dt \quad &= \quad P(1-\rho)^2 b_{0,0}^2 \sum_{g=0}^{q-1} \sum_{m=0}^{q-1} \rho^{g+m} \times \cdots \\ &\underbrace{\int_0^T p_T \left(\frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right) p_T \left(\frac{T}{\left(T - \frac{m}{\beta}\right)} \left(t - \frac{m}{\beta}\right) \right) dt}_{\eta} \end{aligned} \tag{96}$$

Looking at Eq. (96), we have interchanged the order of integration and summation. As the pulse functions are the only functions dependent on time in the expression, we look at the product of the two unit amplitude pulse functions in η



We have arbitrarily assumed that $m > g$, but our logic would also work if we assumed $g > m$. The product of two pulses is 1 for $m/\beta \le t \le T$ and 0 otherwise. Thus, we see that the integral of the product of the two pulse functions in η will be a square area of height $1 \times (T-$ largest of g/β or $m/\beta)$. Thus

$$\int_0^T K^2 |s_B|^2 dt = P(1-\rho)^2 b_{0,0}^2 \sum_{g=0}^{q-1} \sum_{m=0}^{q-1} \rho^{g+m} \left(T - \left(\frac{\max(g,m)}{\beta} \right) \right), \qquad \text{for } 0 \le t \le T \tag{97}$$

where $\max(g,m)$ is defined by

$\max(x_1, x_2)$: Choose largest of x_1 or x_2, which are both positive. If $x_1 = x_2$, then $\max(x_1, x_2) = x_1 = x_2$.

We now need to compute $\int_0^T K^2 |s_{ISI}|^2 dt$. We see from Eq. (78) that s_{ISI} is real and not complex since $b_{0,i} \in \{0,1\}$. Thus

$$s_{ISI} = s_{ISI}^* \qquad \text{and} \qquad |s_{ISI}|^2 = s_{ISI}^2 \tag{98}$$

$$|s_{ISI}|^2 = \Bigg[\underbrace{\sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \rho^g b_{0,i} p_T \left(\frac{\beta T(t)}{g + (1+i)q} \right)}_{a}$$

$$+ \underbrace{\sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \rho^g b_{0,i} p_T \left(\frac{T}{\left(T - \left(\frac{g+iq}{\beta} \right) \right)} \left(t - \left(\frac{g+iq}{\beta} \right) \right) \right)}_{b} \Bigg]^2 \tag{99}$$

To perform the squaring we set up another version of $a+b$ with the g-index changed to m and the i-index changed to r to account for cross-product terms. We call this new representation $c+d$. Thus

$$|s_{ISI}|^2 = (a+b)(c+d) = \underbrace{ac}_{\mathcal{A}} + \underbrace{ad}_{\mathcal{B}} + \underbrace{bc}_{\mathcal{C}} + \underbrace{bd}_{\mathcal{D}}$$

Performing the multiplications yields

$$\begin{aligned} \mathcal{A} = ac \;\; &= \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \rho^{g+m} b_{0,i} b_{0,r} \times \cdots \\ & p_T\left(\frac{\beta T(t)}{g+(1+i)q}\right) p_T\left(\frac{\beta T(t)}{m+(1+r)q}\right) \end{aligned} \tag{100}$$

$$\begin{aligned} \mathcal{B} = ad \;\; &= \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} \times \cdots \\ & p_T\left(\frac{\beta T(t)}{g+(1+i)q}\right) p_T\left(\frac{T\left(t-\left(\frac{m+rq}{\beta}\right)\right)}{\left(T-\left(\frac{m+rq}{\beta}\right)\right)}\right) \end{aligned} \tag{101}$$

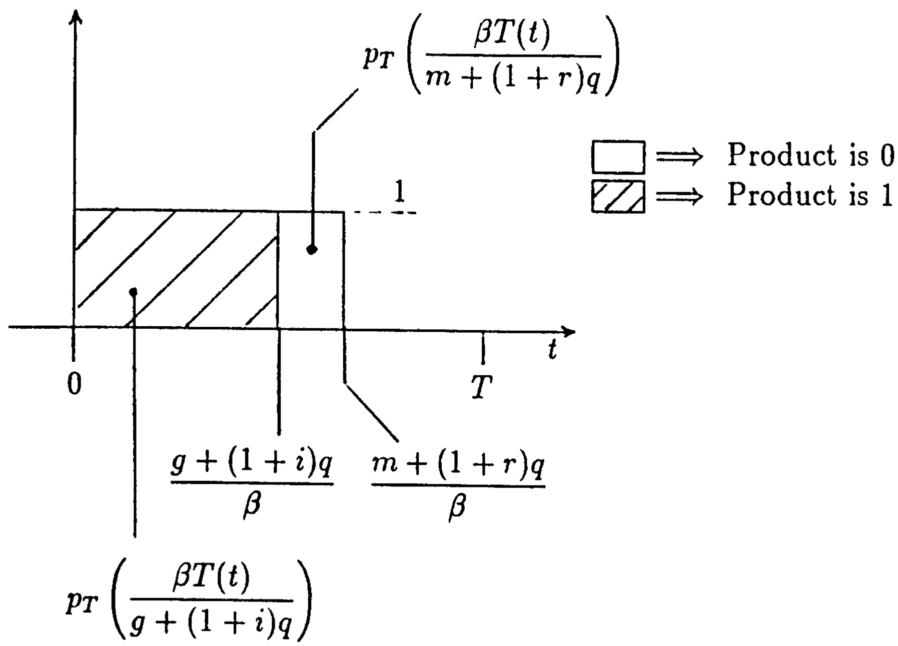
$$\begin{aligned} \mathcal{C} = bc \;\; &= \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{r=-L_0}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \rho^{g+m} b_{0,i} b_{0,r} \times \cdots \\ & p_T\left(\frac{T\left(t-\left(\frac{g+iq}{\beta}\right)\right)}{\left(T-\left(\frac{g+iq}{\beta}\right)\right)}\right) p_T\left(\frac{\beta T(t)}{m+(1+r)q}\right) \end{aligned} \tag{102}$$

$$\begin{aligned} \mathcal{D} = bd \;\; &= \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} \times \cdots \\ & p_T\left(\frac{T\left(t-\left(\frac{g+iq}{\beta}\right)\right)}{\left(T-\left(\frac{g+iq}{\beta}\right)\right)}\right) p_T\left(\frac{T\left(t-\left(\frac{m+rq}{\beta}\right)\right)}{\left(T-\left(\frac{m+rq}{\beta}\right)\right)}\right) \end{aligned} \tag{103}$$

Integrating over the interval of interest: $\int_0^T K^2 |s_{ISI}|^2 dt = \cdots$

$$K^2 \int_0^T |s_{ISI}|^2 dt = K^2 \left[\int_0^T \mathcal{A} dt + \int_0^T \mathcal{B} dt \int_0^T \mathcal{C} dt + \int_0^T \mathcal{D} dt \right] \tag{104}$$

The equations for $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$ are Eqs. (100)–(103), respectively. We now analyze term $\mathcal{A}$ [Eq. (100)] by sketching the multiplication of the pulses



Again, we have arbitrarily assumed one pulse lasts longer than the other. Since the height of each scaled pulse is 1, we see that the area under the product is 1 times the minimum of

$$\frac{q+(1+i)q}{\beta} \quad \text{or} \quad \frac{m+(1+r)q}{\beta}$$

We can apply the same logic as used before to arrive at the value of the term

$$K^2 \int_0^T \mathcal{A}\, dt$$

Then

$$K^2 \int_0^T \mathcal{A}\, dt = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \varphi_0 \tag{105}$$

where

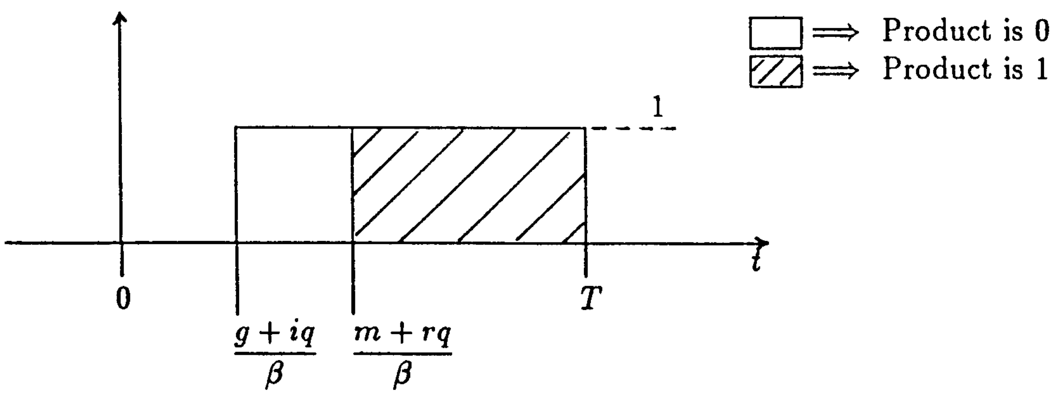
$$\varphi_0 = \rho^{g+m} b_{0,i} b_{0,r} \left(\frac{\min\left(g+(1+i)q, m+(1+r)q\right)}{\beta} \right)$$

where

$$\min\left(g+(1+i)q,\ m+(1+r)q\right) \quad \text{is defined by}$$

$\min(x_1, x_2)$: Choose smallest of x_1 or x_2, which are both positive. If $x_1 = x_2$, then $\min(x_1, x_2) = x_1 = x_2$.

We now compute $K^2 \int_0^T \mathcal{D} dt$. Looking at the expression for $\mathcal{D}$ [Eq. (103)], we sketch the product of the pulses, arbitrarily assuming $m + rq > g + iq$



We see the product of pulses only exists from the maximum of

$$\frac{g+iq}{\beta} \quad \text{or} \quad \frac{m+rq}{\beta}$$

Thus, the integral of the product of the pulses will be

$$T - \frac{\max\left(g+iq,\ m+rq\right)}{\beta}$$

Then

$$K^2 \int_0^T \mathcal{D}\, dt = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} \times \cdots \left[T - \left(\frac{\max(g+iq,\ m+rq)}{\beta} \right) \right] \tag{106}$$

Looking at the expression for $\mathcal{B}$ [Eq. (101)]

$$\mathcal{B} = \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} \times \cdots$$
$$p_T\left(\frac{\beta T(t)}{g+(1+i)q}\right) p_T\left(\frac{T\left(t-\left(\frac{m+rq}{\beta}\right)\right)}{\left(T-\left(\frac{m+rq}{\beta}\right)\right)}\right) \tag{107}$$

We note that m, g, i, and r are simply dummy variables and may be interchanged, giving

$$\mathcal{B} = \sum_{r=-L_0}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \rho^{g+m} b_{0,r} b_{0,i} \times \cdots$$
$$p_T\left(\frac{\beta T(t)}{m+(1+r)q}\right) p_T\left(\frac{T\left(t-\left(\frac{g+iq}{\beta}\right)\right)}{\left(T-\left(\frac{g+iq}{\beta}\right)\right)}\right) \tag{108}$$

If we look at the expression above Eq. (108), we see that $\mathcal{B}$ is identical to $\mathcal{C}$ [Eq. (102)] in every way except the order in which the summations appear. The order of the summations can be rearranged because the terms inside of the summations are completely separable in relation to the sets $\{m,r\}$, $\{i,g\}$. Thus, we can rearrange the order of the summations and conclude

$$\mathcal{B} = \mathcal{C} \tag{109}$$

and that

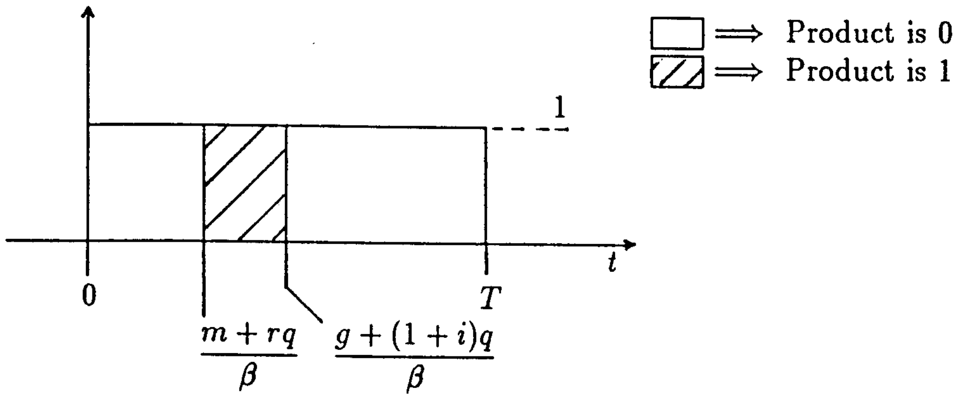
$$K^2 \int_0^T \mathcal{B} dt + K^2 \int_0^T \mathcal{C} dt = 2K^2 \int_0^T \mathcal{B} dt = 2K^2 \int_0^T \mathcal{C} dt \tag{110}$$

We choose to integrate twice the value of $\mathcal{B}$ [Eq. (101)]. Interchanging the order of integration and summation yields

$$2K^2 \int_0^T \mathcal{B}\, dt = 2K^2 \Bigg(\sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} \times \cdots$$

$$\int_0^T p_T\left(\frac{\beta T(t)}{g+(1+i)q}\right) p_T\left(\frac{T\left(t-\left(\frac{m+rq}{\beta}\right)\right)}{\left(T-\left(\frac{m+rq}{\beta}\right)\right)}\right) dt\Big) \quad (111)$$

Sketching the product of the two pulses, we see the product will exist for $m + rq < g + (1+i)q$ and will be zero otherwise.



We create a "gating" function to account for this

$$\int_0^T p_T\left(\frac{\beta T(t)}{g+(1+i)q}\right) p_T\left(\frac{T\left(t-\left(\frac{m+rq}{\beta}\right)\right)}{\left(T-\left(\frac{m+rq}{\beta}\right)\right)}\right) dt =$$

$$G_1(g,i,m,r) = \left\{ \begin{array}{ll} \frac{1}{\beta}[(g+(1+i)q)-(m+rq)], & \text{for } m+rq < g+(1+i)q \\ 0, & \text{otherwise} \end{array} \right\} \quad (112)$$

Then finally

$$\begin{aligned} 2K^2 \int_0^T \mathcal{B} dt &= K^2 \int_0^T \mathcal{B} dt + K^2 \int_0^T \mathcal{C} dt \\ &= 2P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} G_1(g,i,m,r) \end{aligned} \quad (113)$$

Finally, combining the results of Eqs. (105), (106), and (113) we have

$$K^2 \int_0^T s_{ISI} s_{ISI}^* dt = K^2 \int_0^T |s_{ISI}|^2 dt = K^2 \left[\int_0^T \mathcal{A}\, dt + \int_0^T \mathcal{B}\, dt \int_0^T \mathcal{C}\, dt + \int_0^T \mathcal{D}\, dt \right] \tag{114}$$

where

$$K^2 \int_0^T \mathcal{A}\, dt = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \varphi_0$$

in which

$$\varphi_0 = \rho^{g+m} b_{0,i} b_{0,r} \left(\frac{\min(g+(1+i)q, m+(1+r)q)}{\beta} \right)$$

$$K^2 \int_0^T \mathcal{B}\, dt + K^2 \int_0^T \mathcal{C}\, dt = 2K^2 \int_0^T \mathcal{B}\, dt$$

$$= 2P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} G_1(g,i,m,r)$$

and

$$\begin{aligned} K^2 \int_0^T \mathcal{D}\, dt &= P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} \times \cdots \\ & \left[T - \left(\frac{\max(g+iq, m+rq)}{\beta} \right) \right] \end{aligned}$$

Now we must compute $|s_{ACI}|^2$ and $K^2 \int_0^T |s_{ACI}|^2 dt$. We see that we have a complex $b_{k,\ell}$ as $b_{k,\ell} \in \{0, e^{j\phi_k}\}$, as well as a complex exponential $e^{j\omega_k(t-(g/\beta))}$ [see Eq. (83)].

We begin by noting that the complex conjugate operator has the following properties

$$\begin{aligned} (uv)^* &= u^* v^* \\ (u+v)^* &= u^* + v^* \end{aligned}$$

Taking the complex conjugate of Eq. (83)

$$s_{ACI}^* = \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \left(\left[\sum_{g=0}^{q-1} \rho^g b_{k,0}^* e^{-j\omega_k(t-(g/\beta))} p_T \left(\frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right) \right] \right.$$

$$+\sum_{\ell=-L}^{-1}\left[\sum_{g=-(1+\ell)q+1}^{(-\ell q)-1}\rho^g b_{k,\ell}^* e^{-j\omega_k(t-(g/\beta))}p_T\left(\frac{\beta T(t)}{g+(1+\ell)q}\right)\right.$$

$$\left.\left. + \sum_{g=-\ell q}^{-(\ell-1)q-1}\rho^g b_{k,\ell}^* e^{-j\omega_k(t-(g/\beta))}p_T\left(\frac{T\left(t-\left(\frac{g+\ell q}{\beta}\right)\right)}{\left(T-\left(\frac{g+\ell q}{\beta}\right)\right)}\right)\right]\right) \tag{115}$$

Letting

$$\mathrm{a} \;=\; p_T\left(\frac{T}{\left(T-\frac{g}{\beta}\right)}\left(t-\frac{g}{\beta}\right)\right) \tag{116}$$

$$\mathrm{a}' \;=\; p_T\left(\frac{T}{\left(T-\frac{m}{\beta}\right)}\left(t-\frac{m}{\beta}\right)\right) \tag{117}$$

and

$$\mathrm{c} \;=\; p_T\left(\frac{\beta T(t)}{g+(1+\ell)q}\right) \tag{118}$$

$$\mathrm{c}' \;=\; p_T\left(\frac{\beta T(t)}{m+(1+r)q}\right) \tag{119}$$

$$\mathrm{d} \;=\; p_T\left(\frac{T\left(t-\left(\frac{g+\ell q}{\beta}\right)\right)}{\left(T-\left(\frac{g+\ell q}{\beta}\right)\right)}\right) \tag{120}$$

$$\mathrm{d}' \;=\; p_T\left(\frac{T\left(t-\left(\frac{m+rq}{\beta}\right)\right)}{\left(T-\left(\frac{m+rq}{\beta}\right)\right)}\right) \tag{121}$$

Then, with a change of indices, we can express s_{ACI}^* as

$$s_{ACI}^* \;=\; \underbrace{\sum_{\substack{n=-M/2\\ n\neq 0}}^{+M/2}\sum_{m=0}^{q-1}\rho^m b_{n,0}^* e^{-j\omega_n(t-(m/\beta))}\mathrm{a}'}_{\mathrm{A}^*}$$

$$
\underbrace{+ \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \rho^m b_{n,r}^* e^{-j\omega_n(t-(m/\beta))} \mathrm{c}'}_{\mathrm{B}^*}
$$

$$
\underbrace{+ \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^m b_{n,r}^* e^{-j\omega_n(t-(m/\beta))} \mathrm{d}'}_{\mathrm{C}^*} \tag{122}
$$

Now

$$
\begin{aligned}
s_{ACI} = & \underbrace{\sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{g=0}^{q-1} \rho^g b_{k,0} e^{j\omega_k(t-(g/\beta))} \mathrm{a}}_{\mathrm{A}} \\
& \underbrace{+ \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \rho^g b_{k,\ell} e^{j\omega_k(t-(g/\beta))} \mathrm{c}}_{\mathrm{B}} \\
& \underbrace{+ \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-\ell q}^{-(\ell-1)q-1} \rho^g b_{k,\ell} e^{j\omega_k(t-(g/\beta))} \mathrm{d}}_{\mathrm{C}}
\end{aligned} \tag{123}
$$

Then

$$
|s_{ACI}|^2 = s_{ACI} s_{ACI}^* = (\mathrm{A} + \mathrm{B} + \mathrm{C})(\mathrm{A}^* + \mathrm{B}^* + \mathrm{C}^*) \tag{124}
$$

Now, distribution of multiplication over addition yields

$$
\begin{aligned}
|s_{ACI}|^2 = & \mathrm{AA}^* + \mathrm{AB}^* + \mathrm{AC}^* + \mathrm{BA}^* + \mathrm{BB}^* + \mathrm{BC}^* \\
& + \mathrm{CA}^* + \mathrm{CB}^* + \mathrm{CC}^*
\end{aligned} \tag{125}
$$

Rearranging yields

$$|s_{ACI}|^2 = \mathrm{AA}^* + \mathrm{BB}^* + \mathrm{CC}^* + 2\left[\frac{1}{2}\left(\mathrm{AB}^* + \mathrm{BA}^*\right)\right]$$
$$+2\left[\frac{1}{2}\left(\mathrm{AC}^* + \mathrm{CA}^*\right)\right] + 2\left[\frac{1}{2}\left(\mathrm{BC}^* + \mathrm{CB}^*\right)\right] \quad (126)$$

Using the identity $Re[Z] = (Z + Z^*)/2$ we have

$$|s_{ACI}|^2 = \mathrm{AA}^* + \mathrm{BB}^* + \mathrm{CC}^* + 2Re[\mathrm{AC}^*]$$
$$+2Re\left[\mathrm{AB}^*\right] + 2Re[\mathrm{BC}^*] \quad (127)$$

Multiplying the terms above yields

$$\mathrm{AA}^* = \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n\neq 0}}^{+M/2} \sum_{m=0}^{q-1} \rho^{g+m} b_{k,0} b_{n,0}^* e^{j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]}\mathrm{aa}' \quad (128)$$

$$\mathrm{BB}^* = \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \sum_{\substack{n=-M/2 \\ n\neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \gamma \quad (129)$$

where

$$\gamma = \rho^{g+m} b_{k,\ell} b_{n,r}^* e^{j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]}\mathrm{cc}'$$

$$\mathrm{CC}^* = \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-\ell q}^{-(\ell-1)q-1} \sum_{\substack{n=-M/2 \\ n\neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \zeta \quad (130)$$

where

$$\zeta = \rho^{g+m} b_{k,\ell} b_{n,r}^* e^{j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]}\mathrm{dd}'$$

To compute $2Re\left[\mathrm{AC}^*\right]$, we know $Re[\cdot]$ is a linear operator so it may be moved across the summations. After multiplication of terms A and C^* we get

$$2Re\left[\mathrm{AC}^*\right] = 2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n\neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \kappa \quad (131)$$

where

$$\kappa = Re\left[\rho^{g+m} b_{k,0} b_{n,r}^{*} e^{j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]} \mathrm{ad}'\right]$$

We know $b_{k,0}$, $b_{n,r}^{*}$, and $e^{j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]}$ are complex. We will calculate κ in Eq. (131) using the identity

$$Re[Z] = \frac{1}{2}(Z + Z^{*})$$

Since $(uv)^{*} = u^{*}v^{*}$, by extension it can be shown that

$$(uvw)^{*} = u^{*}v^{*}w^{*}$$

Also note that if $S = a_1 + a_2 + a_3 + a_4 + \cdots$ where $a_1, a_2, a_3, a_4, \cdots$ are complex, then

$$Re[S] = Re[a_1] + Re[a_2] + Re[a_3] + Re[a_4] + \cdots$$

We will also use the fact that if z is complex and α is constant, then

$$Re[\alpha z] = \alpha Re[z]$$

Using the facts above, we can see

$$\begin{aligned} \kappa \;&=\; \frac{1}{\cancel{2}}\Big[b_{k,0} b_{n,r}^{*} e^{j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]} \\ &\quad + b_{k,0}^{*} b_{n,r} e^{-j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]}\Big] \rho^{g+m} \mathrm{ad}' \end{aligned} \tag{132}$$

Then

$$\begin{aligned} \kappa \;&=\; \Big[b_{k,0} b_{n,r}^{*} e^{j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]} \\ &\quad + b_{k,0}^{*} b_{n,r} e^{-j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]}\Big] \rho^{g+m} \mathrm{ad}' \end{aligned} \tag{133}$$

with a and d′ defined by Eqs. (116) and (121), respectively.

Now, we will turn to computing $2Re\,[\mathrm{AB}^*]$. This will have five summations just like the $2Re\,[\mathrm{AC}^*]$ term above.

$$2Re\,[\mathrm{AB}^*] = \sum_{\substack{k=-M/2\\k\neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2\\n\neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \varsigma \tag{134}$$

By inspection of Eq. (123)

$$\varsigma = Re\left[\rho^{g+m} b_{k,0} b_{n,r}^{*} e^{j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]}\mathrm{ac}'\right]$$

After letting the 1/2 generated when applying $Re[\cdot]$ operator to the argument cancel the 2 outside of the summations, we get

$$\begin{aligned}\varsigma \;=\; & \Big[b_{k,0} b_{n,r}^{*} e^{j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]} \\ & +b_{k,0}^{*} b_{n,r} e^{-j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]}\Big]\, \rho^{g+m}\mathrm{ac}' \end{aligned} \tag{135}$$

With a and c′ defined by Eqs. (116) and (119), respectively.

We now compute the term $2Re[\mathrm{BC}^*]$. This term will have six summations. Moving the $Re[\cdot]$ operator inside of the six summations to the argument yields the following by inspection

$$\iota = Re\left[b_{k,\ell} b_{n,r}^{*} e^{j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]}\rho^{g+m}\mathrm{cd}'\right] \tag{136}$$

After letting the 1/2 generated when applying the $Re[\cdot]$ operator in ι [Eq. (136)] cancel the 2 outside of the summations, we can then write down the expression for $2Re[\mathrm{BC}^*]$

$$2Re\,[\mathrm{BC}^*] \;=\; \sum_{\substack{k=-M/2\\k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \sum_{\substack{n=-M/2\\n\neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \iota \tag{137}$$

$$\begin{aligned}\iota \;=\; & \Big[b_{k,\ell} b_{n,r}^{*} e^{j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]} + b_{k,\ell}^{*} b_{n,r} \times \cdots \\ & e^{-j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]}\Big]\, \rho^{g+m}\mathrm{cd}'\end{aligned}$$

with c and d′ defined by Eqs. (118) and (121).

Now we must find

$$\int_0^T K^2|s_{ACI}|^2dt = K^2\int_0^T \mathrm{AA}^*dt + K^2\int_0^T \mathrm{BB}^*dt + K^2\int_0^T \mathrm{CC}^*dt$$
$$+K^2\int_0^T 2Re\left[\mathrm{AC}^*\right]dt + K^2\int_0^T 2Re\left[\mathrm{AB}^*\right]dt$$
$$+K^2\int_0^T 2Re\left[\mathrm{BC}^*\right]dt \qquad (138)$$

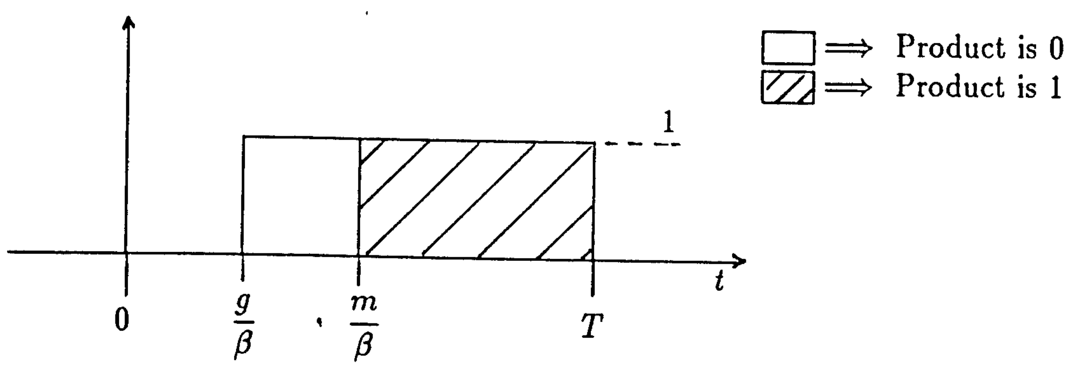
where we recall that

$$K^2 = P(1-\rho)^2$$

In the term AA* [Eq. (128)], we have the pulse multiplication aa′ where

$$\mathrm{a} = p_T\left(\frac{T\left(t-\frac{g}{\beta}\right)}{\left(T-\frac{g}{\beta}\right)}\right) \qquad \mathrm{a}' = p_T\left(\frac{T\left(t-\frac{m}{\beta}\right)}{\left(T-\frac{m}{\beta}\right)}\right)$$

Arbitrarily assuming $m > g$, we sketch the product a × a′.



The product is 1 and exists from the maximum of $(m/\beta, g/\beta)$ to T. Either g or m can be the largest. Hence, we will be integrating

$$\xi = e^{j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]} \qquad (139)$$

with the lower limit of integration being $(1/\beta)\max(g,m)$. The upper limit of integration is T. We consider two cases for Eq. (128).

Case 1: $k \neq n \Longrightarrow \omega_k \neq \omega_n$

When $k \neq n$, $\omega_k \neq \omega_n$ because

$$\omega_k = k\Delta\omega \tag{140}$$

where again ω_k is the radian frequency spacing between Channel 0 and Channel k, and $\Delta\omega$ is the uniform radian frequency spacing between adjacent channels.

$$\begin{aligned} K^2 \int_0^T \mathrm{AA}^* dt &= P(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n\neq 0}}^{+M/2} \sum_{m=0}^{q-1} \rho^{g+m} b_{k,0} b_{n,0}^* \times \cdots \\ & \int_{(1/\beta)\max(g,m)}^{T} \xi dt \end{aligned} \tag{141}$$

Letting

$$\begin{aligned} x &= j\left[(\omega_k - \omega_n)t - \omega_k\,(g/\beta) + \omega_n\,(m/\beta)\right] \\ dx &= j\left[(\omega_k - \omega_n)\right] dt \end{aligned}$$

then

$$\begin{aligned} \int_{(1/\beta)\max(g,m)}^{T} \xi dt &= \frac{1}{j(\omega_k - \omega_n)} e^{j[(\omega_k-\omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \Big|_{(1/\beta)\max(g,m)}^{T} \\ &= \frac{1}{j(\omega_k - \omega_n)} \Big[e^{j[(\omega_k-\omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} \\ & \quad - e^{j[(\omega_k-\omega_n)(1/\beta)\max(g,m) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \Big] \end{aligned} \tag{142}$$

Case 2: $k = n \Longrightarrow \omega_k = \omega_n$

Again similar logic to that used above [Eq. (140)] allows us to conclude $\omega_k = \omega_n$. For this case, the complex exponential ξ reduces to

$$\xi = e^{j[0 - \omega_k(g/\beta) + \omega_n(m/\beta)]} \tag{143}$$

Now, since $\omega_k = \omega_n$, we can further reduce the complex exponential to

$$\xi = e^{j[(\omega_k/\beta)(m-g)]} \tag{144}$$

Therefore, for $\omega_k = \omega_n$

$$\int_{(1/\beta)\max(g,m)}^{T} \xi dt = e^{j[(\omega_k/\beta)(m-g)]} \int_{(1/\beta)\max(g,m)}^{T} dt = e^{j[(\omega_k/\beta)(m-g)]} \left[T - \frac{1}{\beta} \max(g, m) \right] \tag{145}$$

Thus, we may write the expression for $K^2 \int_0^T \mathrm{AA}^* dt$

$$K^2 \int_0^T \mathrm{AA}^* dt = P(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \rho^{g+m} \times \varphi_1 \tag{146}$$

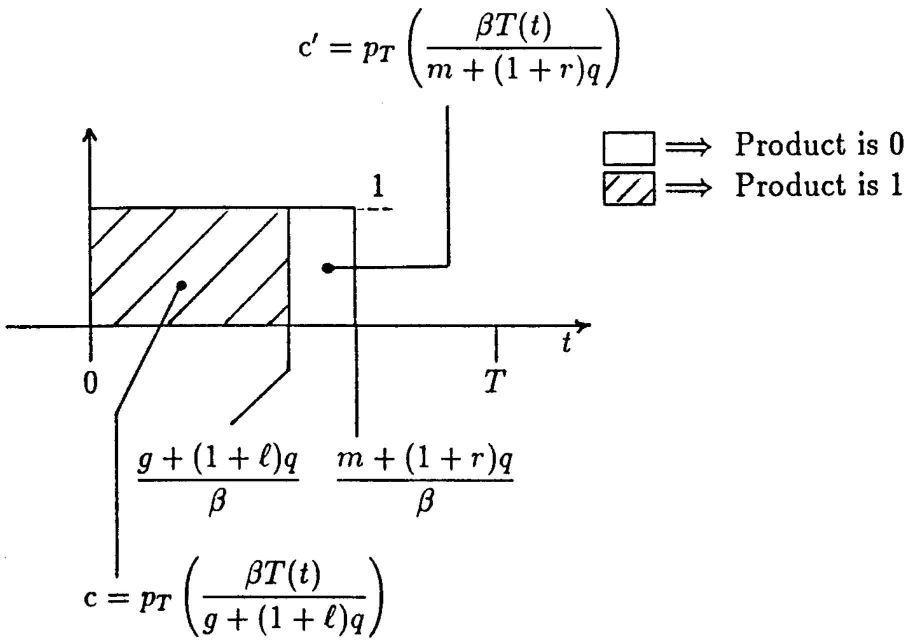
where

$$\varphi_1 = \left\{ \begin{array}{ll} b_{k,0} b_{n,0}^* \dfrac{1}{j(\omega_k - \omega_n)} \left[e^{j[(\omega_k - \omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. & \\ \left. - e^{j[(\omega_k - \omega_n)(1/\beta)\max(g,m) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right], & \text{for } k \neq n \\ \underbrace{b_{k,0} b_{k,0}^*}_{|b_{k,0}|^2} e^{j[(\omega_k/\beta)(m-g)]} \left[T - \dfrac{1}{\beta} \max(g, m) \right], & \text{for } k = n \end{array} \right\}$$

Now we turn to the BB^* term [Eq. (129)]. We have the pulse multiplication cc′ where

$$\mathrm{c} = p_T \left(\frac{\beta T(t)}{g + (1+\ell)q} \right) \qquad \mathrm{c}' = p_T \left(\frac{\beta T}{m + (1+r)q} \right)$$

Arbitrarily assuming $m + (1+r)q > g + (1+\ell)q$, we sketch the product $\mathrm{c} \times \mathrm{c}'$.



The product exists and is 1 from 0 to minimum of

$$\left(\frac{g+(1+\ell)q}{\beta}, \frac{m+(1+r)q}{\beta}\right)$$

Hence, we will be integrating the same complex exponential ξ [Eq. (139)] as for the AA* term [Eq. (128)], but the upper limit of integration will be

$$\frac{1}{\beta}\min(g+(1+\ell)q,\ m+(1+r)q)$$

while the lower limit is 0. We again consider two cases.

Case 1: $k \neq n \Longrightarrow \omega_k \neq \varphi_n$

After interchanging the order of integration and summation, we will be integrating [see Eq. (129)]

$$\xi = e^{j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]} \tag{147}$$

Then

$$\int_0^{(1/\beta)\min(g+(1+\ell)q,m+(1+r)q)} \xi dt$$

$$= \frac{1}{j(\omega_k - \omega_n)} e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \Big|_0^{(1/\beta)\min(g+(1+\ell)q, m+(1+r)q)}$$

$$= \frac{1}{j(\omega_k - \omega_n)} \Big[e^{j[(\omega_k-\omega_n)(1/\beta)\min(g+(1+\ell)q, m+(1+r)q) - \omega_k(g/\beta) + \omega_n(m/\beta)]}$$

$$- e^{j[-\omega_k(g/\beta)+\omega_n(m/\beta)]} \Big] \tag{148}$$

Then, factoring the result yields

$$\int_0^{(1/\beta)\min(g+(1+\ell)q, m+(1+r)q)} \xi dt =$$

$$\frac{1}{j(\omega_k - \omega_n)} e^{j[-\omega_k(g/\beta)+\omega_n(m/\beta)]} \left[e^{j[(\omega_k-\omega_n)(1/\beta)\min(g+(1+\ell)q, m+(1+r)q)]} - 1 \right] \tag{149}$$

Case 2: $k = n \Longrightarrow \omega_k = \omega_n$

This is similar to the AA* term for $\omega_k = \omega_n$ [see Eq. (144)]

$$\xi = e^{j[(\omega_k/\beta)(m-g)]} \tag{150}$$

and

$$\int_0^{(1/\beta)\min(g+(1+\ell)q, m+(1+r)q)} \xi \, dt = e^{j[(\omega_k/\beta)(m-g)]} \int_0^{(1/\beta)\min(g+(1+\ell)q, m+(1+r)q)} dt$$

$$= e^{j[(\omega_k/\beta)(m-g)]} \left[\frac{1}{\beta} \min(g + (1+\ell)q, m + (1+r)q) - 0 \right] \tag{151}$$

Thus, we may write the expression for $K^2 \int_0^T \mathrm{BB}^* dt$

$$K^2 \int_0^T \mathrm{BB}^* dt = P(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \sum_{\substack{n=-M/2 \\ n\neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \rho^{g+m} \times \varphi_2 \tag{152}$$

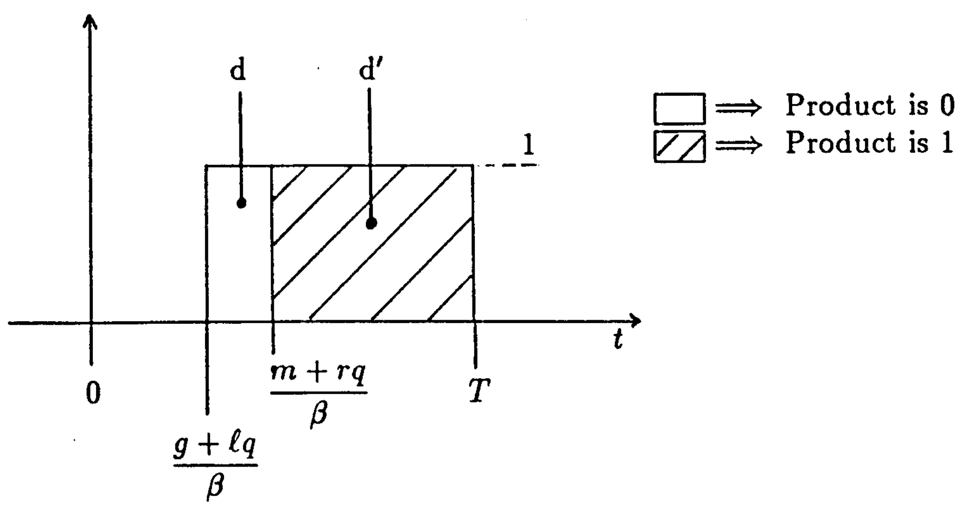
where

$$
\varphi_2 = \left\{ \begin{array}{l} b_{k,\ell} b^*_{n,r} \left[\dfrac{e^{j[-\omega_k(g/\beta)+\omega_n(m/\beta)]}}{j(\omega_k - \omega_n)} \left(e^{j[(\omega_k-\omega_n)(1/\beta)\min(g+(1+\ell)q, m+(1+r)q)]} - 1 \right) \right], \\ \hfill \text{for } k \neq n \\ \\ = b_{k,\ell} b^*_{k,r} \left(e^{j[(\omega_k/\beta)(m-g)]} \left[\dfrac{1}{\beta} \min(g + (1+\ell)q, m + (1+r)q) \right] \right), \\ \hfill \text{for } k = n \end{array} \right\}
$$

Now we integrate the CC* term [Eq. (130)]. We have the pulse multiplication dd′ where

$$
\mathrm{d} = p_T \left(\frac{T\left(t - \left(\frac{g+\ell q}{\beta}\right)\right)}{\left(T - \left(\frac{g+\ell q}{\beta}\right)\right)} \right) \qquad \mathrm{d}' = p_T \left(\frac{T\left(t - \left(\frac{m+rq}{\beta}\right)\right)}{\left(T - \left(\frac{m+rq}{\beta}\right)\right)} \right)
$$

Looking at the pulse multiplication graphically where we arbitrarily assume $m + rq > g + \ell q$ for the purpose of sketching the situation, we have



Looking at the equation for CC* [Eq. (130)], we see that after interchanging the order of integration and summation, we will be integrating the same complex exponential as for the AA* and BB* terms

$$
\xi = e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \tag{153}
$$

where the lower limit of integration will be

$$\frac{1}{\beta}\max(g+\ell q, m+rq)$$

while the upper limit will be T. Then for

Case 1: $k \neq n \Longrightarrow \omega_k \neq \omega_n$

$$\begin{aligned}
\int_{(1/\beta)\max(g+\ell q,m+rq)}^{T} \xi\, dt &= \frac{1}{j(\omega_k-\omega_n)} e^{j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]}\Big|_{(1/\beta)\max(g+\ell q,m+rq)}^{T} \\
&= \frac{1}{j(\omega_k-\omega_n)}\Big[e^{j[(\omega_k-\omega_n)T-\omega_k(g/\beta)+\omega_n(m/\beta)]} \\
&\quad -e^{j[(\omega_k-\omega_n)(1/\beta)\max(g+\ell q,m+rq)-\omega_k(g/\beta)+\omega_n(m/\beta)]}\Big] \qquad (154)
\end{aligned}$$

Case 2: $k = n \Longrightarrow \omega_k = \omega_n$

Again, similar to the AA* term, for $k = n$, the term

$$\xi = e^{j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]} \qquad (155)$$

reduces to

$$\xi = e^{j[(\omega_k/\beta)(m-g)]} \qquad (156)$$

and then

$$\begin{aligned}
\int_{(1/\beta)\max(g+\ell q,m+rq)}^{T} \xi\, dt &= e^{j[(\omega_k/\beta)(m-g)]}\int_{(1/\beta)\max(g+\ell q,m+rq)}^{T} dt \\
&= e^{j[(\omega_k/\beta)(m-g)]}\left[T-\frac{1}{\beta}\max(g+\ell q,m+rq)\right] \qquad (157)
\end{aligned}$$

Thus we may write the expression for $K^2\int_0^T \mathrm{CC}^* dt$

$$K^2\int_0^T \mathrm{CC}^* dt = P(1-\rho)^2 \sum_{\substack{k=-M/2\\k\neq 0}}^{+M/2}\ \sum_{\ell=-L}^{-1}\ \sum_{g=-\ell q}^{-(\ell-1)q-1}\ \sum_{\substack{n=-M/2\\n\neq 0}}^{+M/2}\ \sum_{r=-L}^{-1}\ \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m}\times\varphi_3 \qquad (158)$$

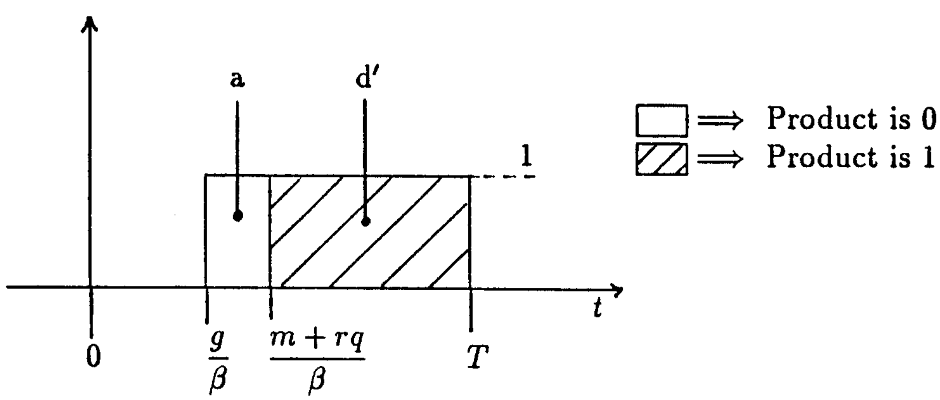
where

$$\varphi_3 = \left\{ \begin{array}{ll} b_{k,\ell}b^*_{n,r}\left[\frac{1}{j(\omega_k-\omega_n)}\left[e^{j[(\omega_k-\omega_n)T-\omega_k(g/\beta)+\omega_n(m/\beta)]}\right.\right. & \\ \left.\left.-e^{j[(\omega_k-\omega_n)(1/\beta)\max(g+\ell q,m+rq)-\omega_k(g/\beta)+\omega_n(m/\beta)]}\right]\right], & \text{for } k\neq n \\ b_{k,\ell}b^*_{k,r}e^{j[(\omega_k/\beta)(m-g)]}\left[T-\frac{1}{\beta}\max(g+\ell q,m+rq)\right], & \text{for } k=n \end{array} \right\}$$

Now we turn to the $2Re\,[\mathrm{AC}^*]$ term [Eq. (131)]. We have the pulse multiplication of ad′ where

$$\mathrm{a} = p_T\left(\frac{T\left(t-\frac{g}{\beta}\right)}{\left(T-\frac{g}{\beta}\right)}\right) \qquad \mathrm{d}' = p_T\left(\frac{T\left(t-\left(\frac{m+rq}{\beta}\right)\right)}{\left(T-\left(\frac{m+rq}{\beta}\right)\right)}\right)$$

Looking at this graphically by arbitrarily assuming $m+rq > g$, we sketch the product a × d′



Then we can see that the product ad′ will exist from $(1/\beta)\max(g,m+rq)$ to T, which are the lower and upper limits of integration, respectively.

Case 1: $k\neq n \Longrightarrow \omega_k\neq\omega_n$

This expression has the two complex exponentials

$$\xi \;=\; e^{j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]} \tag{159}$$

$$\varrho = e^{-j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]} \tag{160}$$

to integrate. Letting

$$\mathrm{u} = -j\left[(\omega_k-\omega_n)t-\omega_k\left(g/\beta\right)+\omega_n\left(m/\beta\right)\right]$$

and

$$d\mathrm{u} = -j(\omega_k-\omega_n)$$

the integration yields

$$\begin{aligned}\int_{(1/\beta)\max(g,m+rq)}^{T} \varrho\, dt &= \frac{1}{[-j(\omega_k-\omega_n)]} e^{-j[(\omega_k-\omega_n)t-\omega_k(g/\beta)+\omega_n(m/\beta)]}\Bigg]_{(1/\beta)\max(g,m+rq)}^{T} \\ &= \frac{1}{[-j(\omega_k-\omega_n)]}\Big[e^{-j[(\omega_k-\omega_n)T-\omega_k(g/\beta)+\omega_n(m/\beta)]} \\ &\quad -e^{-j[(\omega_k-\omega_n)(1/\beta)\max(g,m+rq)-\omega_k(g/\beta)+\omega_n(m/\beta)]}\Big]\end{aligned} \tag{161}$$

and

$$\begin{aligned}\int_{(1/\beta)\max(g,m+rq)}^{T} \xi\, dt &= \frac{1}{j(\omega_k-\omega_n)}\Big[e^{j[(\omega_k-\omega_n)T-\omega_k(g/\beta)+\omega_n(m/\beta)]} \\ &\quad -e^{j[(\omega_k-\omega_n)(1/\beta)\max(g,m+rq)-\omega_k(g/\beta)+\omega_n(m/\beta)]}\Big]\end{aligned} \tag{162}$$

Case 2: $k = n \Longrightarrow \omega_k = \omega_n$

Looking at the expression for $2Re\,[\mathrm{AC}^*]$ [Eq. (131)], we see that for $\omega_k = \omega_n$ the two complex exponentials reduce to $e^{j[(\omega_k/\beta)(m-g)]}$ and $e^{-j[(\omega_k/\beta)(m-g)]}$, which are constants to the integration. Thus, we will integrate the product of the pulse functions ad' from 0 to T. This is the only time function for this case. Then, looking at the sketch below Eq. (158)

$$\int_0^T ad'dt = \int_{(1/\beta)\max(g,m+rq)}^{T} dt = T - \frac{1}{\beta}\max(g, m+rq) \tag{163}$$

Thus, we are ready to write the expression for

$$K^2\int_0^T 2Re\,[\mathrm{AC}^*]\, dt$$

Then,

$$K^2 \int_0^T 2Re\,[\mathrm{AC}^*]\,dt = P(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n\neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \varphi_4 \tag{164}$$

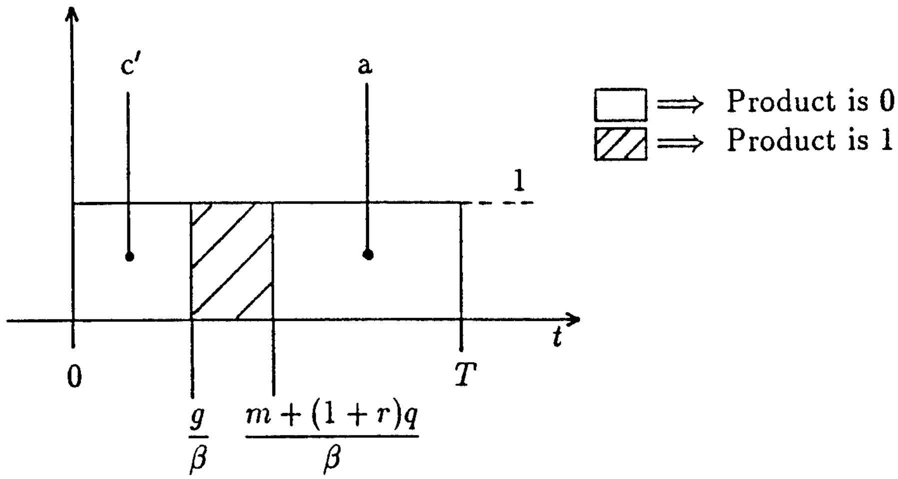
where

$$\varphi_4 = \begin{cases} \rho^{g+m} b_{k,0} b^*_{n,r} \left[\dfrac{1}{j(\omega_k-\omega_n)} \left[e^{j[(\omega_k-\omega_n)T-\omega_k(g/\beta)+\omega_n(m/\beta)]}\right.\right. \\ \left.\left. -e^{j[(\omega_k-\omega_n)(1/\beta)\max(g,m+rq)-\omega_k(g/\beta)+\omega_n(m/\beta)]}\right]\right] \\ +\rho^{g+m} b^*_{k,0} b_{n,r} \left[\frac{1}{[-j(\omega_k-\omega_n)]} \left[e^{-j[(\omega_k-\omega_n)T-\omega_k(g/\beta)+\omega_n(m/\beta)]}\right.\right. \\ \left.\left. -e^{-j[(\omega_k-\omega_n)(1/\beta)\max(g,m+rq)-\omega_k(g/\beta)+\omega_n(m/\beta)]}\right]\right], & \text{for } k\neq n \\ \left[b_{k,0} b^*_{k,r} e^{j[(\omega_k/\beta)(m-g)]} + b^*_{k,0} b_{k,r} e^{-j[(\omega_k/\beta)(m-g)]}\right] \rho^{g+m} \times \cdots \\ \left[T - \dfrac{1}{\beta}\max(g, m+rq)\right], & \text{for } k=n \end{cases}$$

We now turn to the $2Re\,[\mathrm{AB}^*]$ term [Eq. (134)]. We have the pulse multiplication ac$'$ where

$$\mathrm{a} = p_T\left(\frac{T}{\left(T-\dfrac{g}{\beta}\right)}\left(t-\frac{g}{\beta}\right)\right) \qquad \mathrm{c}' = p_T\left(\frac{\beta T(t)}{m+(1+r)q}\right)$$

To form a sketch, we arbitrarily assume $m+(1+r)q > g$, yielding



We see that the product will exist for $m + (1 + r)q > g$ and will be 0 otherwise. Thus, unless $m + (1 + r)q > g$, the integral of $2Re\,[\mathrm{AB}^*]$ is 0. We will later create a "gating" function to account for this.

Case 1: $k \neq n \Longrightarrow \omega_k \neq \omega_n$

For this case where $m + (1 + r)q > g$ we will be integrating the same complex exponentials as before

$$\xi = e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \tag{165}$$

and

$$\varrho = e^{-j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \tag{166}$$

We have computed these integrals before and will just substitute the new upper and lower limits in the final answer for this term

$$\text{Upper Limit} : \quad \frac{(m + (1 + r)q)}{\beta}$$

$$\text{Lower Limit} : \quad \frac{g}{\beta}$$

If $m + (1 + r)q < g$, the integrals of the two complex exponentials are zero as the product of the pulse functions $ac' = 0$. Again, we will create a "gating" function to allow for this.

Case 2: $k = n \Longrightarrow \omega_k = \omega_n$

Looking at the $2Re\,[\mathrm{AB}^*]$ term [Eq. (134)]. We see that the two complex exponentials reduce to

$$e^{j[(\omega_k/\beta)(m-g)]} \quad \text{and} \quad e^{-j[(\omega_k/\beta)(m-g)]}$$

These are constants to the integration with respect to t. Thus as before, we will only be integrating the product of the pulse functions ac$'$ from 0 to T. The integral has value if $m+(1+r)q > g$ and is 0 otherwise. Then, if $m+(1+r)q > g$, the limits of integration are the same as for the case $\omega_k \neq \omega_n$ and

$$\int_0^T \mathrm{ac}'dt = \int_{g/\beta}^{(m+(1+r)q)/\beta} dt = \frac{1}{\beta}[(m+(1+r)q)-g] \tag{167}$$

Now we will create the "gating" function to null the integrals if the product ac$'$ is 0 from 0 to T.

To indicate the zeroing/nulling of all integrals if $g \geq m+(1+r)q$, we will create another gating function called $G_2(g,m,r)$.

$$G_2(g,m,r) = \left\{ \begin{array}{ll} 1, & \text{for } g < m+(1+r)q \\ & \\ 0, & \text{otherwise} \end{array} \right\} \tag{168}$$

We are now ready to write the expression for $K^2 \int_0^T 2Re\,[\mathrm{AB}^*]\,dt$. Then

$$K^2 \int_0^T 2Re\,[\mathrm{AB}^*]\,dt = P(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n\neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \varphi_5 \tag{169}$$

where

$$\varphi_5 = \left\{ \begin{array}{ll} \Big[\Big(\rho^{g+m} b_{k,0} b^*_{n,r} \frac{1}{j(\omega_k-\omega_n)} \Big[e^{j[(\omega_k-\omega_n)((m+(1+r)q)/\beta)-\omega_k(g/\beta)+\omega_n(m/\beta)]} & \\ -e^{j[(\omega_n/\beta)(m-g)]}\Big]\Big) & \\ +\Big(\rho^{g+m} b^*_{k,0} b_{n,r} \frac{1}{[-j(\omega_k-\omega_n)]} \Big[e^{-j[(\omega_k-\omega_n)((m+(1+r)q)/\beta)-\omega_k(g/\beta)+\omega_n(m/\beta)]} & \\ -e^{-j[(\omega_n/\beta)(m-g)]}\Big]\Big)\Big] G_2(g,m,r), & \text{for } k \neq n \\ \Big[b_{k,0} b^*_{k,r} e^{j[(\omega_k/\beta)(m-g)]} + b^*_{k,0} b_{k,r} e^{-j[(\omega_k/\beta)(m-g)]}\Big] \rho^{g+m} \times \cdots & \\ \left(\frac{1}{\beta}[(m+(1+r)q)-g]\right) G_2(g,m,r), & \text{for } k = n \end{array} \right\}$$

where

$$G_2(g,m,r) = \left\{ \begin{array}{ll} 1, & \text{for } g < m+(1+r)q \\ 0, & \text{otherwise} \end{array} \right\}$$

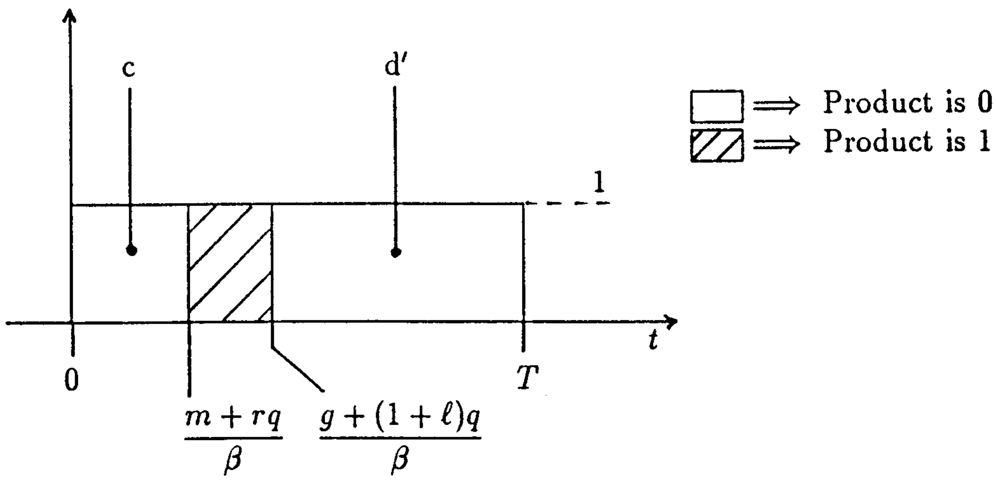
Now, finally we must write out the last (sixth) term of the ACI. We now need to find

$$K^2 \int_0^T 2Re\,[\mathrm{BC}^*]\, dt$$

Again, we look at the expression for $2Re\,[\mathrm{BC}^*]$ [Eq. (137)] to see the pulse multiplication cd$'$ where

$$\mathrm{c} = p_T\left(\frac{\beta T(t)}{g+(1+\ell)q}\right) \qquad \mathrm{d}' = p_T\left(\frac{T\left(t-\left(\frac{m+rq}{\beta}\right)\right)}{\left(T-\left(\frac{m+rq}{\beta}\right)\right)}\right)$$

We sketch at this pulse multiplication by arbitrarily assuming $g+(1+\ell)q > m+rq$.



Case 1: $k \neq n \Longrightarrow \omega_k \neq \omega_n$

Here we will be integrating the complex exponentials

$$\xi = e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \tag{170}$$

and

$$\varrho = e^{-j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \tag{171}$$

The integrals will have the following limits

$$\text{Upper}: \quad \frac{g+(1+\ell)q}{\beta}$$

$$\text{Lower}: \quad \frac{m+rq}{\beta}$$

The integrals will have value for $m + rq < g + (1+\ell)q$ and will be zero otherwise. We have computed these integrals before and will just substitute the new limits in the final answer for this term. We will create another "gating" function to account for the integrals "turning off" when $m + rq \geq g + (1+\ell)q$

$$G_3(g,\ell,m,r) = \left\{ \begin{array}{ll} 1, & m+rq < g+(1+\ell)q \\ 0, & \text{otherwise} \end{array} \right\} \tag{172}$$

Case 2: $k = n \Longrightarrow \omega_k = \omega_n$

Looking at the $2Re\,[\mathrm{BC}^*]$ term [Eq. (137)], we see again that the complex exponentials reduce to

$$e^{j[(\omega_k/\beta)(m-g)]} \quad \text{and} \quad e^{-j[(\omega_k/\beta)(m-g)]}$$

which are constants to the integration with respect to t. Thus, in a similar manner as before, we will only be integrating the product of the pulse functions cd$'$ from 0 to T. The integral has value if $m + rq < g + (1+\ell)q$ and is zero otherwise. We have already created the "gating" function [Eq. (172)] to account for this turning on and off of the integral. Then, if $m + rq < g + (1+\ell)q$ the limits of integration are the same as for the case $\omega_k \neq \omega_n$ and

$$\int_0^T \mathrm{cd}' dt = \int_{(m+rq)/\beta}^{(g+(1+\ell)q)/\beta} dt = \frac{1}{\beta}[(g + (1+\ell)q) - (m + rq)] \tag{173}$$

We are now ready to write the expression for

$$K^2 \int_0^T 2Re\,[\mathrm{BC}^*]\,dt\,.$$

Then

$$K^2 \int_0^T 2Re\,[\mathrm{BC}^*]\,dt = P(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \sum_{\substack{n=-M/2 \\ n\neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \varphi_6 \tag{174}$$

where

$$\varphi_6 = \begin{cases} \Big(\rho^{g+m} b_{k,\ell} b^*_{n,r} \frac{1}{j(\omega_k-\omega_n)} \Big[e^{j[(\omega_k-\omega_n)((g+(1+\ell)q)/\beta)-\omega_k(g/\beta)+\omega_n(m/\beta)]} \\ -e^{j[(\omega_k-\omega_n)((m+rq)/\beta)-\omega_k(g/\beta)+\omega_n(m/\beta)]}\Big] \\ +\rho^{g+m} b^*_{k,\ell} b_{n,r} \dfrac{1}{[-j(\omega_k-\omega_n)]} \Big[e^{-j[(\omega_k-\omega_n)((g+(1+\ell)q)/\beta)-\omega_k(g/\beta)+\omega_n(m/\beta)]} \\ -e^{-j[(\omega_k-\omega_n)((m+rq)/\beta)-\omega_k(g/\beta)+\omega_n(m/\beta)]}\Big]\Big) G_3(g,\ell,m,r), & \text{for } k \neq n \\ \Big[b_{k,\ell} b^*_{k,r} e^{j[(\omega_k/\beta)(m-g)]} + b^*_{k,\ell} b_{k,r} e^{-j[(\omega_k/\beta)(m-g)]}\Big] \rho^{g+m} \times \cdots \\ \left(\dfrac{1}{\beta}[(g+(1+\ell)q)-(m+rq)]\right) G_3(g,\ell,m,r), & \text{for } k = n \end{cases}$$

where $G_3(g,\ell,m,r)$ is defined by Eq. (172). We have now completed integrating all six terms of the ACI.

Recalling the expression for $|s(t)|^2$ [Eq. (89)], we see that we still have to compute the following terms

$$2Re[s_B s^*_{ISI}] = 2Re[s_{ISI} s^*_B],$$

$$2Re[s^*_B s_{ACI}] = 2Re[s_B s^*_{ACI}],$$

and

$$2Re[s^*_{ISI} s_{ACI}] = 2Re[s_{ISI} s^*_{ACI}],$$

multiply each by the constant $K^2 = P(1-\rho)^2$ and integrate each from 0 to T. We will use either form of the above three expressions depending on the ease of computation. Recall from Eq. (66) that

$$s_B(t) = \sum_{g=0}^{q-1} \rho^g b_{0,0} p_T\left(\frac{T}{\left(T-\frac{g}{\beta}\right)}\left(t-\frac{g}{\beta}\right)\right), \qquad \text{for } 0 \le t \le T \qquad (175)$$

Let us look at $s_B s^*_{ISI}$ term. Looking at the expression for $s_B(t)$ during the interval $0 \le t \le T$ [Eqs. (66) or (175)], we see that $s_B(t)$ is real as $b_{0,i} \in \{0,1\}$. Looking at the expression for $s_{ISI}(t)$ during the interval $0 \leq t \leq T$ [Eq. (78)], we see also that $s_{ISI}(t)$ is real as again $b_{0,i} \in \{0,1\}$. Then $s_{ISI} = s^*_{ISI}$ and $s_B s^*_{ISI} = s_B s_{ISI}$, and

$$2Re[s_B s^*_{ISI}] = 2 s_B s_{ISI} \tag{176}$$

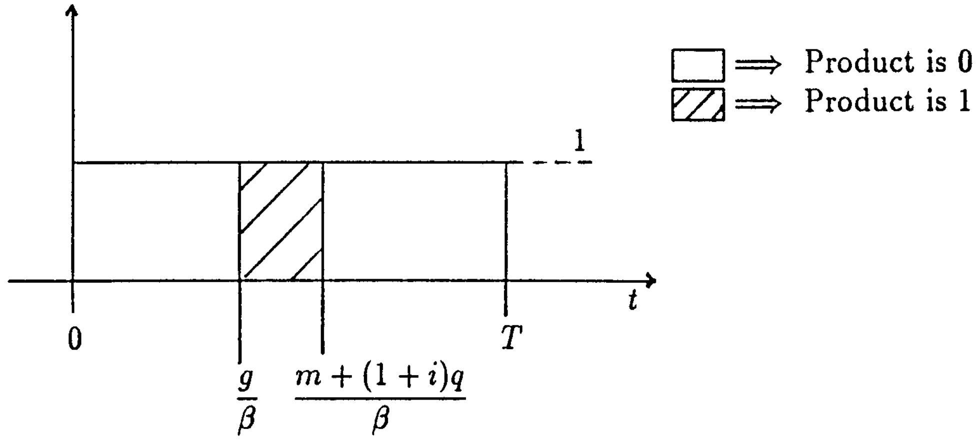
Performing the multiplication yields

$$\begin{aligned}
2Re[s_B s^*_{ISI}] &= 2[s_B s_{ISI}] \\
&\underbrace{= 2\sum_{g=0}^{q-1}\sum_{i=-L_0}^{-1}\sum_{m=-(1+i)q+1}^{(-iq)-1} \rho^{g+m} b_{0,0} b_{0,i} p_T\left(\frac{T}{\left(T-\frac{g}{\beta}\right)}\left(t-\frac{g}{\beta}\right)\right) p_T\left(\frac{\beta T(t)}{m+(1+i)q}\right)}_{\mathrm{e}} \\
&\underbrace{+2\sum_{g=0}^{q-1}\sum_{i=-L_0}^{-1}\sum_{m=-iq}^{-(i-1)q-1} \rho^{g+m} b_{0,0} b_{0,i} p_T\left(\frac{T\left(t-\frac{g}{\beta}\right)}{\left(T-\frac{g}{\beta}\right)}\right) p_T\left(\frac{T\left(t-\left(\frac{m+iq}{\beta}\right)\right)}{\left(T-\left(\frac{m+iq}{\beta}\right)\right)}\right)}_{\mathrm{f}}
\end{aligned} \tag{177}$$

We see that when we multiply by K^2 and integrate, the integral will distribute across addition to the two terms e and f. The integrals then move inside of the summations to the pulse function products. Now we need to figure the proper limits for the integration of the two pulse function products in e and f [Eq. (177)]. We note that the products of pulse functions are the only time functions in the expression $2Re[s_B s^*_{ISI}]$, so they are the only integrands in the expression. We examine the pulse multiplication in term e [Eq. (177)]. Let

$$\mathrm{E} = p_T\left(\frac{T\left(t-\frac{g}{\beta}\right)}{\left(T-\frac{g}{\beta}\right)}\right) p_T\left(\frac{\beta T(t)}{m+(1+i)q}\right) \tag{178}$$

Sketching E, we arbitrarily assume $m + (1+i)q > g$.



The integral $\int_0^T \mathrm{E}\,dt$ will have value for $g < m + (1+i)q$ and will be zero otherwise. If $g < m + (1+i)q$

$$\int_0^T \mathrm{E}\,dt = \int_{g/\beta}^{(m+(1+i)q)/\beta} dt = \frac{1}{\beta}\left[(m + (1+i)q) - g\right] \tag{179}$$

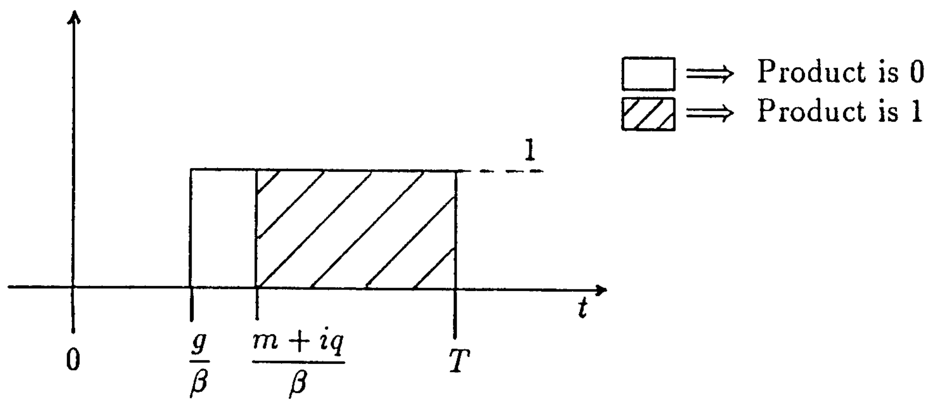
Since only the pulse function product is being integrated, we will create another gating function, which also gives us the value of the above integral when it turns on. This is similar to $G_1(g, i, m, r)$ [Eq. (112)].

$$\int_0^T \mathrm{E}\,dt = G_4(g, i, m) = \left\{ \begin{array}{cc} \frac{1}{\beta}\left[(m + (1+i)q) - g\right], & g < m + (1+i)q \\ 0, & \text{otherwise} \end{array} \right\} \tag{180}$$

Look at the pulse multiplication in term f [Eq. (177)]. Let

$$\mathrm{F} = p_T\left(\frac{T\left(t - \frac{g}{\beta}\right)}{\left(T - \frac{g}{\beta}\right)}\right) p_T\left(\frac{T\left(t - \left(\frac{m+iq}{\beta}\right)\right)}{\left(T - \left(\frac{m+iq}{\beta}\right)\right)}\right) \tag{181}$$

Sketching F, we arbitrarily assume $m + iq > g$.



We see that

$$\int_0^T \mathrm{F}\,dt = T - \frac{1}{\beta}\max(g, m + iq) \tag{182}$$

We can now write down the following integral

$$K^2 \int_0^T 2Re[s_B s_{ISI}^*]dt$$

Substituting integrals $\int_0^T \mathrm{E}\,dt$ and $\int_0^T \mathrm{F}\,dt$ for the pulse function products in the $2Re[s_B s_{ISI}^*]$ expression [Eq. (177)], and multiplying by K^2 yields the following result.

Note:

$\int_0^T \mathrm{E}\,dt$ is substituted for the pulse function product in e in Eq. (177).

$\int_0^T \mathrm{F}\,dt$ is substituted for the pulse function product in f in Eq. (177).

Recalling that $K^2 = P(1-\rho)^2$ we have

$$K^2 \int_0^T 2Re[s_B s_{ISI}^*]dt = K^2 \int_0^T 2[s_B s_{ISI}]dt$$

$$= 2P(1-\rho)^2 \sum_{g=0}^{q-1} \sum_{i=-L_0}^{-1} \sum_{m=-(1+i)q+1}^{(-iq)-1} \rho^{g+m} b_{0,0} b_{0,i} G_4(g,i,m)$$

$$+ 2P(1-\rho)^2 \sum_{g=0}^{q-1} \sum_{i=-L_0}^{-1} \sum_{m=-iq}^{-(i-1)q-1} \rho^{g+m} b_{0,0} b_{0,i} \left[T - \frac{1}{\beta}\max(g, m+iq)\right] \tag{183}$$

where

$$G_4(g,i,m) = \begin{Bmatrix} \dfrac{1}{\beta}\left[(m+(1+i)q)-g\right], & g < m+(1+i)q \\ 0, & \text{otherwise} \end{Bmatrix}$$

Now we need to compute $2Re[s_B^* s_{ACI}]$, multiply by K^2, and integrate it from 0 to T. We note again that $s_B = s_B^*$ thus $s_B^* s_{ACI} = s_B s_{ACI}$ and since s_{ACI} is complex we will distribute the $Re[\cdot]$ operator over the summations of the $s_B s_{ACI}$ product in the same manner as in some of the previous terms.

Computing the $2Re[s_B^* s_{ACI}] = 2Re[s_B s_{ACI}]$ term

$$2Re[s_B^* s_{ACI}] = 2Re[s_B s_{ACI}] = 2Re\left[\sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{m=0}^{q-1} \rho^{g+m} b_{0,0} b_{k,0} e^{j\omega_k(t-(m/\beta))} \times \cdots\right.$$

$$\underbrace{p_T\left(\frac{T}{\left(T-\frac{g}{\beta}\right)}\left(t-\frac{g}{\beta}\right)\right) p_T\left(\frac{T}{\left(T-\frac{m}{\beta}\right)}\left(t-\frac{m}{\beta}\right)\right)}_{\text{G}}$$

$$+\sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \rho^{g+m} b_{0,0} b_{k,\ell} e^{j\omega_k(t-(m/\beta))} \times \cdots$$

$$\underbrace{p_T\left(\frac{T\left(t-\frac{g}{\beta}\right)}{\left(T-\frac{g}{\beta}\right)}\right) p_T\left(\frac{\beta T(t)}{m+(1+\ell)q}\right)}_{\text{H}}$$

$$+\sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \rho^{g+m} b_{0,0} b_{k,\ell} e^{j\omega_k(t-(m/\beta))} \times \cdots$$

$$\left.\underbrace{p_T\left(\frac{T\left(t-\frac{g}{\beta}\right)}{\left(T-\frac{g}{\beta}\right)}\right)p_T\left(\frac{T\left(t-\left(\frac{m+\ell q}{\beta}\right)\right)}{\left(T-\left(\frac{m+\ell q}{\beta}\right)\right)}\right)}_{\mathrm{I}}\right] \tag{184}$$

The pulse products are G, H, and I as shown above. Then we note again that $b_{0,i}$ is real. Also, $b_{k,\ell} \in \{0, e^{j\phi_k}\}$ and $e^{j\omega_k(t-(m/\beta))}$ are complex. We then have

$$2Re[s_B^* s_{ACI}] = 2Re[s_B s_{ACI}] =$$

$$\begin{aligned}
&\sum_{g=0}^{q-1}\sum_{\substack{k=-M/2\\k\neq 0}}^{+M/2}\sum_{m=0}^{q-1}\left(\rho^{g+m}b_{0,0}b_{k,0}e^{j\omega_k(t-(m/\beta))}\mathrm{G}+\rho^{g+m}b_{0,0}b_{k,0}^*e^{-j\omega_k(t-(m/\beta))}\mathrm{G}\right)\\
&+\sum_{g=0}^{q-1}\sum_{\substack{k=-M/2\\k\neq 0}}^{+M/2}\sum_{\ell=-L}^{-1}\sum_{m=-(1+\ell)q+1}^{(-\ell q)-1}\left(\rho^{g+m}b_{0,0}b_{k,\ell}e^{j\omega_k(t-(m/\beta))}\mathrm{H}\ +\rho^{g+m}b_{0,0}b_{k,\ell}^*e^{-j\omega_k(t-(m/\beta))}\mathrm{H}\right)\\
&+\sum_{g=0}^{q-1}\sum_{\substack{k=-M/2\\k\neq 0}}^{+M/2}\sum_{\ell=-L}^{-1}\sum_{m=-\ell q}^{-(\ell-1)q-1}\left(\rho^{g+m}b_{0,0}b_{k,\ell}e^{j\omega_k(t-(m/\beta))}\mathrm{I}\ +\rho^{g+m}b_{0,0}b_{k,\ell}^*e^{-j\omega_k(t-(m/\beta))}\mathrm{I}\right)
\end{aligned} \tag{185}$$

We see that computing $K^2 \int_0^T 2Re[s_B^* s_{ACI}]$ will involve integrating each of the six terms inside of the three clusters of summations. The integrable part of each of these six terms is a complex exponential of the form

$$\vartheta = e^{j\omega_k(t-(m/\beta))} \tag{186}$$

or

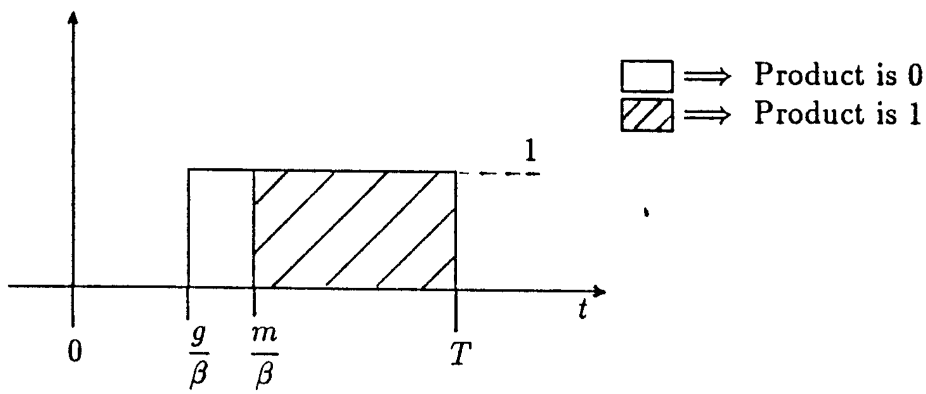
$$\mu = e^{-j\omega_k(t-(m/\beta))} \tag{187}$$

The pulse products G, H, and I will provide the limits of integration.

Looking at G [Eq. (184)]

$$G = p_T\left(\frac{T\left(t-\frac{g}{\beta}\right)}{\left(T-\frac{g}{\beta}\right)}\right) p_T\left(\frac{T\left(t-\frac{m}{\beta}\right)}{\left(T-\frac{m}{\beta}\right)}\right) \tag{188}$$

Sketching G, we arbitrarily assume $m > g$.



Since the height of the pulse product G is 1 from $(1/\beta)\max(g,m)$ to T, we see that we will have the following limits of integration for the complex exponentials

$$\vartheta = e^{j\omega_k(t-(m/\beta))} \tag{189}$$

and

$$\mu = e^{-j\omega_k(t-(m/\beta))} \tag{190}$$

which are the integrands in the first cluster of summations in the $2Re[s_B^* s_{ACI}]$ term [Eq. (185)]

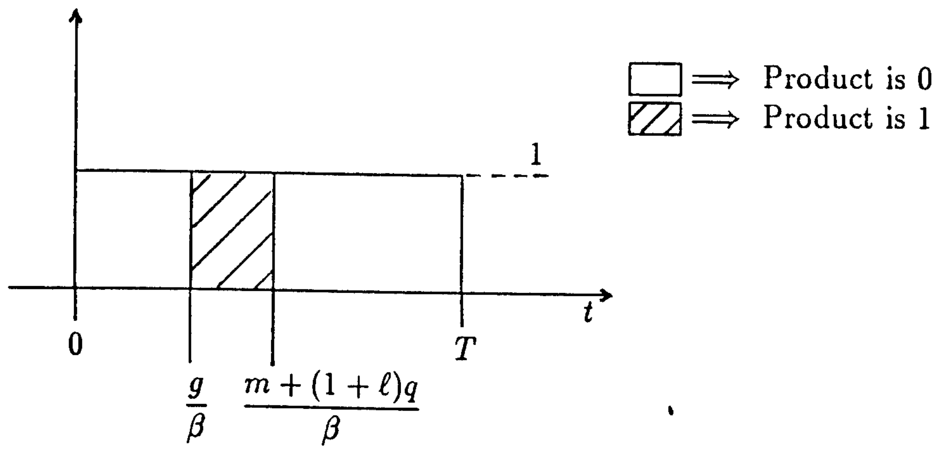
$$\text{Upper Limit}: \quad T$$

$$\text{Lower Limit}: \quad \frac{1}{\beta}\max(g,m)$$

Turning to H [Eq. (184)]

$$H = p_T\left(\frac{T\left(t-\frac{g}{\beta}\right)}{\left(T-\frac{g}{\beta}\right)}\right) p_T\left(\frac{\beta T(t)}{m+(1+\ell)q}\right) \tag{191}$$

Sketching H, we arbitrarily assume $m + (1 + \ell)q > g$.



We see that for $g < m + (1 + \ell)q$, the upper and lower limits of integration for the integrands $\vartheta = e^{j\omega_k(t-(m/\beta))}$ and $\mu = e^{-j\omega_k(t-(m/\beta))}$ contained in the second cluster of summations in the $2Re[s_B^* s_{ACI}]$ term [Eq. (185)] will be

$$\text{Upper Limit :} \quad \frac{1}{\beta}(m + (1 + \ell)q)$$

$$\text{Lower Limit :} \quad \frac{g}{\beta}$$

We will now create another "gating" function to null the integrals of $\vartheta = e^{j\omega_k(t-(m/\beta))}$ and $\mu = e^{-j\omega_k(t-(m/\beta))}$ in the second cluster of summations in the $2Re[s_B^* s_{ACI}]$ term [Eq. (185)]. This nulling of the two integrals will occur if $g \geq m + (1 + \ell)q$.

So we define

$$G_5(g, \ell, m) = \left\{ \begin{array}{ll} 1, & g < m + (1 + \ell)q \\ 0, & \text{otherwise} \end{array} \right\} \tag{192}$$

Now we compute the integrals of $\vartheta = e^{j\omega_k(t-(m/\beta))}$ and $\mu = e^{-j\omega_k(t-(m/\beta))}$ for arbitrary upper and lower limits of integration γ_1 and α_1

$$\int_{\alpha_1}^{\gamma_1} \vartheta dt = \int_{\alpha_1}^{\gamma_1} e^{j\omega_k(t-(m/\beta))} dt = \frac{1}{j\omega_k} e^{j\omega_k(t-(m/\beta))} \Big|_{\alpha_1}^{\gamma_1} \tag{193}$$

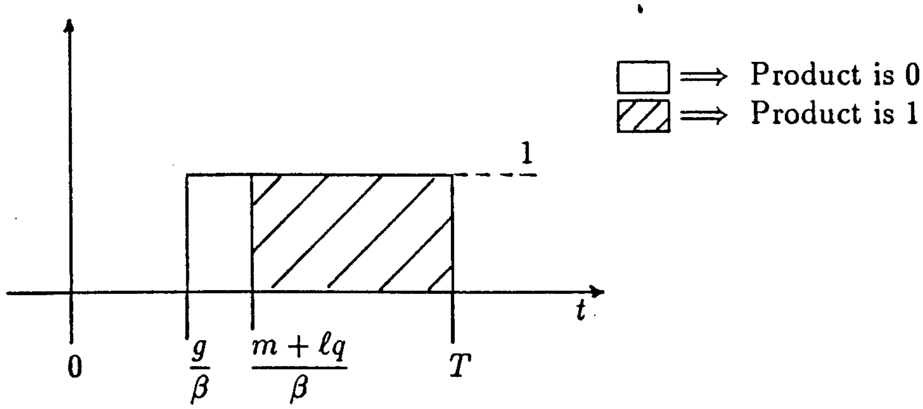
Similarly for μ

$$\int_{\alpha_1}^{\gamma_1} \mu dt = \int_{\alpha_1}^{\gamma_1} e^{-j\omega_k(t-(m/\beta))} dt = \frac{1}{[-j\omega_k]} e^{-j\omega_k(t-(m/\beta))} \Big|_{\alpha_1}^{\gamma_1} \tag{194}$$

Now we turn to the pulse product term I in Eq (184)

$$\mathrm{I} = p_T \left(\frac{T\left(t - \frac{g}{\beta}\right)}{\left(T - \frac{g}{\beta}\right)} \right) p_T \left(\frac{T\left(t - \left(\frac{m+\ell q}{\beta}\right)\right)}{\left(T - \left(\frac{m+\ell q}{\beta}\right)\right)} \right) \tag{195}$$

Sketching I, we arbitrarily assume $m + \ell q > g$.



We see from the sketch that the upper and lower limits of integration of the integrands $\vartheta = e^{j\omega_k(t-(m/\beta))}$ and $\mu = e^{-j\omega_k(t-(m/\beta))}$ in the third cluster of summations in the $2Re[s_B^* s_{ACI}]$ term [Eq. (185)] are

$$\begin{aligned} \text{Upper Limit :} &\quad T \\ \text{Lower Limit :} &\quad \frac{1}{\beta}\max(g, m+\ell q) \end{aligned}$$

Now that we have the three sets of limits of integration for each of the clusters of summations and the indefinite integrals of ϑ and μ, we are prepared to write the expression for

$$K^2 \int_0^T 2Re[s_B^* s_{ACI}]dt = K^2 \int_0^T 2Re[s_B s_{ACI}]dt$$

Then, recalling that $K^2 = P(1-\rho)^2$

$$K^2 \int_0^T 2Re[s_B^* s_{ACI}]dt = K^2 \int_0^T 2Re[s_B s_{ACI}]dt = A_{BA} + B_{BA} + C_{BA} \tag{196}$$

where

$$A_{BA} = P(1-\rho)^2 \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{m=0}^{q-1} \varphi_7$$

in which

$$\begin{aligned} \varphi_7 &= \rho^{g+m} b_{0,0} b_{k,0} \frac{1}{j\omega_k} \left[e^{j\omega_k(T-(m/\beta))} - e^{j\omega_k((\max(g,m)-m)/\beta)} \right] \\ &+ \rho^{g+m} b_{0,0} b_{k,0}^* \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k(T-(m/\beta))} - e^{-j\omega_k((\max(g,m)-m)/\beta)} \right] \end{aligned}$$

$$B_{BA} = P(1-\rho)^2 \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \varphi_8$$

in which

$$\begin{aligned} \varphi_8 &= \rho^{g+m} b_{0,0} b_{k,\ell} \frac{1}{j\omega_k} \left[e^{j\omega_k((1+\ell)q/\beta)} - e^{j\omega_k((g-m)/\beta)} \right] G_5(g,\ell,m) \\ &+ \rho^{g+m} b_{0,0} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k((1+\ell)q/\beta)} - e^{-j\omega_k((g-m)/\beta)} \right] G_5(g,\ell,m) \end{aligned}$$

where

$$G_5(g,\ell,m) = \left\{ \begin{array}{ll} 1, & g < m + (1+\ell)q \\ 0, & \text{otherwise} \end{array} \right\}$$

$$C_{BA} = P(1-\rho)^2 \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \varphi_9$$

in which

$$\varphi_9 = \rho^{g+m} b_{0,0} b_{k,\ell} \frac{1}{j\omega_k} \left[e^{j\omega_k (T-(m/\beta))} - e^{j\omega_k ((\max(g,m+\ell q)-m)/\beta)} \right]$$
$$+\rho^{g+m} b_{0,0} b^*_{k,\ell} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k (T-(m/\beta))} - e^{-j\omega_k ((\max(g,m+\ell q)-m)/\beta)} \right]$$

The last term we have to compute is $K^2 \int_0^T 2Re[s^*_{ISI} s_{ACI}]dt$. We have already shown that s_{ISI} is real; thus

$$2Re[s^*_{ISI} s_{ACI}] = 2Re[s_{ISI} s_{ACI}] \tag{197}$$

As before, we will multiply s_{ISI} and s_{ACI}.

Look at s_{ISI} [Eq. (78)], we will let the first term be A and the second be B. Looking at s_{ACI} [Eq. (83)], we will let the first, second, and third terms be C, D, and E, respectively. Then

$$s_{ISI} s_{ACI} = (A+B)(C+D+E) = AC + AD + AE + BC + BD + BE \tag{198}$$

Then, we compute all six of the above terms individually

$$AC = \sum_{i=L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{m=0}^{q-1} \rho^{g+m} b_{0,i} b_{k,0} e^{j\omega_k (t-(m/\beta))} \times \cdots$$

$$p_T\left(\frac{\beta T(t)}{g+(1+i)q}\right) p_T\left(\frac{T\left(t-\frac{m}{\beta}\right)}{\left(T-\frac{m}{\beta}\right)}\right) \tag{199}$$

$$AD = \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \rho^{g+m} b_{0,i} b_{k,\ell} e^{j\omega_k (t-(m/\beta))} \times \cdots$$

$$p_T\left(\frac{\beta T(t)}{g+(1+i)q}\right) p_T\left(\frac{\beta T(t)}{m+(1+\ell)q}\right) \tag{200}$$

$$AE = \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \rho^{g+m} b_{0,i} b_{k,\ell} e^{j\omega_k(t-(m/\beta))} \times \cdots$$

$$p_T\left(\frac{\beta T(t)}{g+(1+i)q}\right) p_T\left(\frac{T\left(t-\left(\frac{m+\ell q}{\beta}\right)\right)}{\left(T-\left(\frac{m+\ell q}{\beta}\right)\right)}\right) \tag{201}$$

$$BC = \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \rho^{g+m} b_{0,i} b_{k,0} e^{j\omega_k(t-(m/\beta))} \times \cdots$$

$$p_T\left(\frac{T\left(t-\left(\frac{g+iq}{\beta}\right)\right)}{\left(T-\left(\frac{g+iq}{\beta}\right)\right)}\right) p_T\left(\frac{T\left(t-\frac{m}{\beta}\right)}{\left(T-\frac{m}{\beta}\right)}\right) \tag{202}$$

$$BD = \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \rho^{g+m} b_{0,i} b_{k,\ell} e^{j\omega_k(t-(m/\beta))} \times \cdots$$

$$p_T\left(\frac{T\left(t-\left(\frac{g+iq}{\beta}\right)\right)}{\left(T-\left(\frac{g+iq}{\beta}\right)\right)}\right) p_T\left(\frac{\beta T(t)}{m+(1+\ell)q}\right) \tag{203}$$

$$BE = \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \rho^{g+m} b_{0,i} b_{k,\ell} e^{j\omega_k(t-(m/\beta))} \times \cdots$$

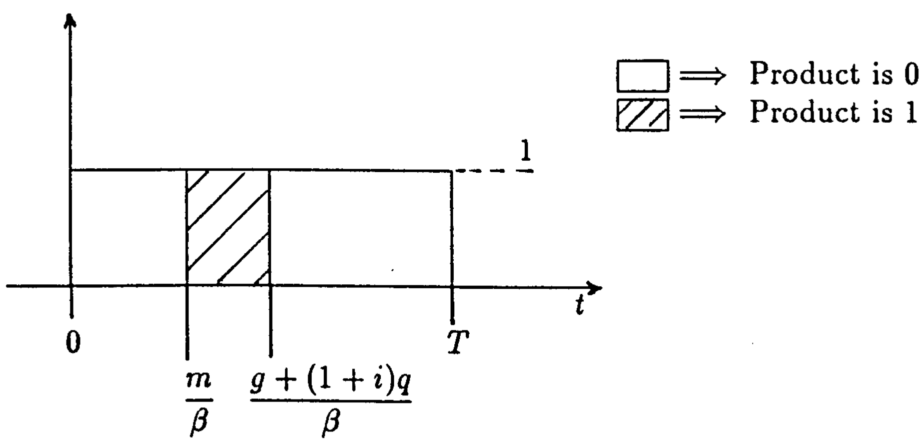
$$p_T\left(\frac{T\left(t-\left(\frac{g+iq}{\beta}\right)\right)}{\left(T-\left(\frac{g+iq}{\beta}\right)\right)}\right) p_T\left(\frac{T\left(t-\left(\frac{m+\ell q}{\beta}\right)\right)}{\left(T-\left(\frac{m+\ell q}{\beta}\right)\right)}\right) \tag{204}$$

When we distribute $2Re[\cdot]$ across each of the terms $AC + AD + \cdots$ and use $Re[Z] = [Z + Z^*]^2$, the 2 will cancel with the 1/2, and we will be integrating the same complex exponential terms $\vartheta = e^{j\omega_k(t-(m/\beta))}$ and $\mu = e^{-j\omega_k(t-(m/\beta))}$ as we did before for the $2Re[s_B^* s_{ACI}]$ term [see Eqs. (186) and (187)]. Then, all we need to do is investigate the pulse products for each of the six terms $AC + AD + \cdots$ to figure out the proper limits of integration. Also, we may need to create other "gating" functions to null the integrals if there are specific index occurences which make the specific pulse product 0.

The AC pulse product is

$$p_T\left(\frac{\beta T(t)}{g + (1 + i)q}\right) p_T\left(\frac{T\left(t - \frac{m}{\beta}\right)}{\left(T - \frac{m}{\beta}\right)}\right)$$

Arbitrarily assuming $g + (1 + i)q > m$, we sketch the pulse product



We see that the limits of integration will be

$$\text{Upper Limit}: \quad \frac{1}{\beta}(g + (1 + i)q)$$

$$\text{Lower Limit}: \quad \frac{m}{\beta}$$

Now we define the appropriate "gating" function

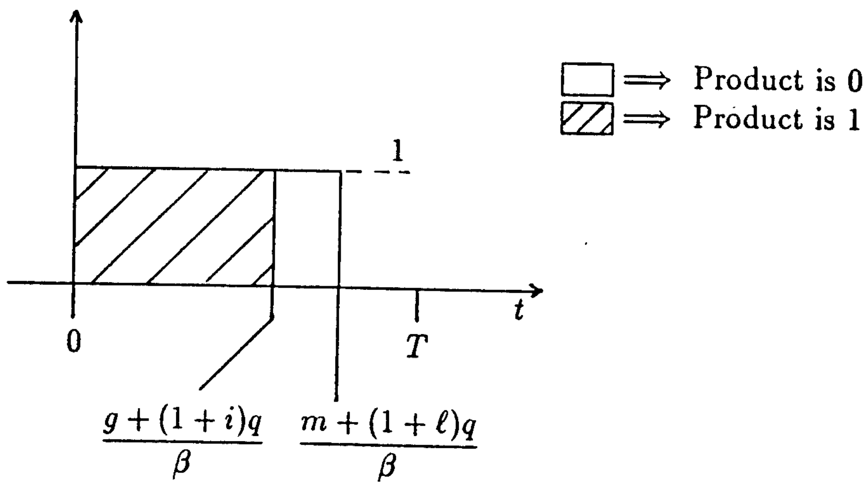
$$G_6(g,i,m) = \left\{ \begin{array}{ll} 1, & m < g + (1+i)q \\ 0, & \text{otherwise} \end{array} \right\} \tag{205}$$

This nulls the two applicable integrals when the pulses do not overlap.

The AD pulse product is

$$p_T\left(\frac{\beta T(t)}{g+(1+i)q}\right) p_T\left(\frac{\beta T(t)}{m+(1+\ell)q}\right)$$

Arbitrarily assuming $m + (1+\ell)q > g + (1+i)q$, we sketch the pulse product.



We see that the limits of integration will be

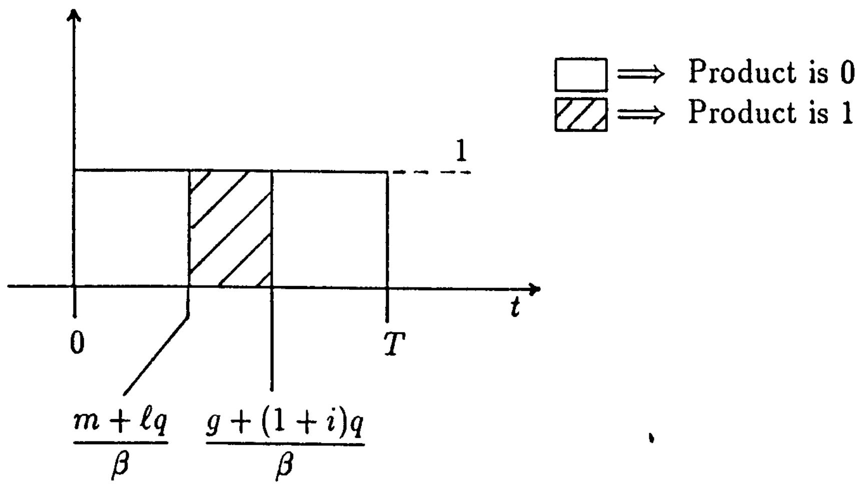
Upper Limit : $\frac{1}{\beta}\min(g+(1+i)q,\ m+(1+\ell)q)$

Lower Limit : 0

The AE pulse product is

$$p_T\left(\frac{\beta T(t)}{g+(1+i)q}\right) p_T\left(\frac{T\left(t-\left(\frac{m+\ell q}{\beta}\right)\right)}{\left(T-\left(\frac{m+\ell q}{\beta}\right)\right)}\right)$$

Arbitrarily assuming $g + (1+i)q > m + \ell q$, we sketch the pulse product.



We see that the limits of integration will be

$$\text{Upper Limit}: \quad \frac{1}{\beta}(g + (1+i)q)$$

$$\text{Lower Limit}: \quad \frac{1}{\beta}(m + \ell q)$$

Now we define an appropriate gating function

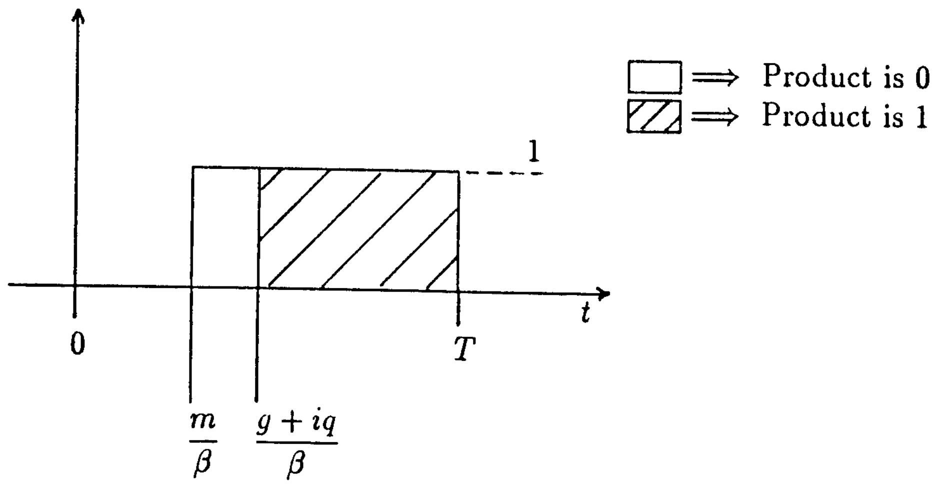
$$G_7(g,i,\ell,m) = \left\{ \begin{array}{ll} 1, & m + \ell q < g + (1+i)q \\ & \\ 0, & \text{otherwise} \end{array} \right\} \tag{206}$$

This nulls the two applicable integrals when the pulses do not overlap.

The BC pulse product is

$$p_T\left(\frac{T\left(t - \left(\frac{g+iq}{\beta}\right)\right)}{\left(T - \left(\frac{g+iq}{\beta}\right)\right)}\right) p_T\left(\frac{T\left(t - \frac{m}{\beta}\right)}{\left(T - \frac{m}{\beta}\right)}\right)$$

Arbitrarily assuming $g + iq > m$, we sketch the pulse product.



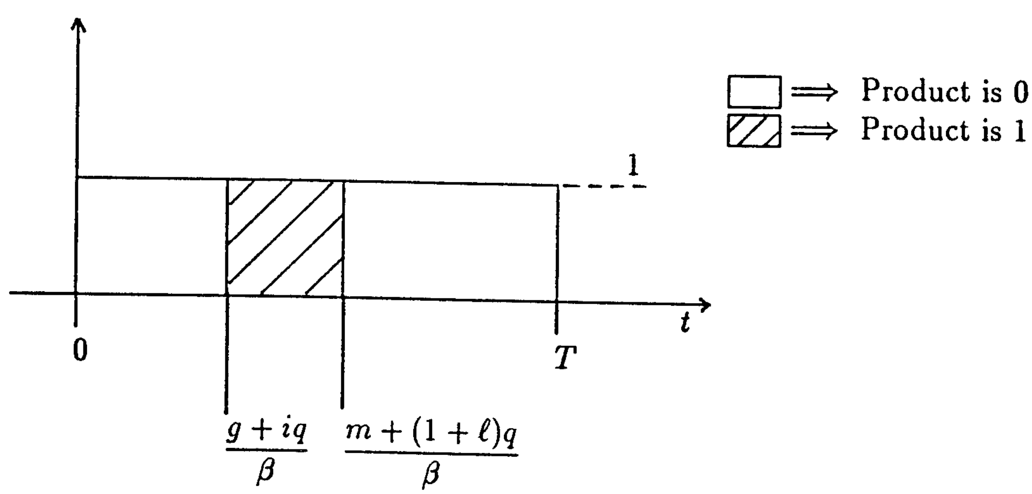
We see that the limits of integration will be

$$\begin{aligned}&\text{Upper Limit :} \quad && T\\ &\text{Lower Limit :} \quad && \frac{1}{\beta}\max(g+iq,m)\end{aligned}$$

The BD pulse product is

$$p_T\left(\frac{T\left(t-\left(\frac{g+iq}{\beta}\right)\right)}{\left(T-\left(\frac{g+iq}{\beta}\right)\right)}\right)p_T\left(\frac{\beta T(t)}{m+(1+\ell)q}\right)$$

Arbitrarily assuming $g+iq<m+(1+\ell)q$, we sketch the pulse product.



We see that the limits of integration will be

$$\text{Upper Limit :} \quad \frac{1}{\beta}(m + (1+\ell)q)$$

$$\text{Lower Limit :} \quad \frac{1}{\beta}(g + iq)$$

We now define an appropriate gating function

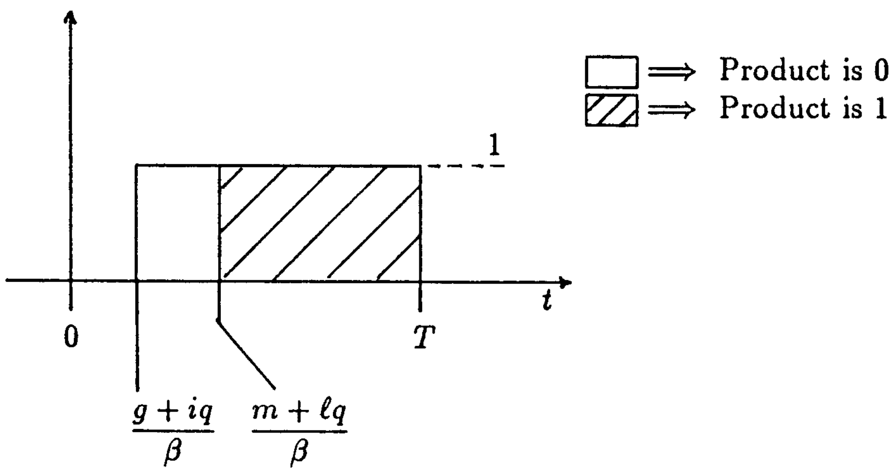
$$G_8(g,i,\ell,m) = \left\{ \begin{array}{ll} 1, & g + iq < m + (1+\ell)q \\ & \\ 0, & \text{otherwise} \end{array} \right\} \tag{207}$$

This nulls the two applicable integrals when the pulses do not overlap.

The BE pulse product is

$$p_T\left(\frac{T\left(t-\left(\frac{g+iq}{\beta}\right)\right)}{\left(T-\left(\frac{g+iq}{\beta}\right)\right)}\right) p_T\left(\frac{T\left(t-\left(\frac{m+\ell q}{\beta}\right)\right)}{\left(T-\left(\frac{m+\ell q}{\beta}\right)\right)}\right)$$

Arbitrarily assuming $m + \ell q > g + iq$, we sketch the pulse product.



We see that the limits of integration will be

$$\text{Upper Limit :} \quad T$$

$$\text{Lower Limit :} \quad \frac{1}{\beta}\max(g + iq, m + \ell q)$$

We will now apply the methods outlined below Eq. (204) and perform the $2Re[\cdot]$ operation on each of the six terms of $s^*_{ISI}s_{ACI} = s_{ISI}s_{ACI}$. We will then integrate the resulting $\vartheta = e^{j\omega_k(t-(m/\beta))}$ and $\mu = e^{-j\omega_k(t-(m/\beta))}$ terms in each of the six clusters of summations and multiply by the proper gating function, if necessary, to compute

$$K^2 \int_0^T 2Re[s^*_{ISI}s_{ACI}]dt = K^2 \int_0^T 2Re[s_{ISI}s_{ACI}]dt$$

Recalling that $K^2 = P(1-\rho)^2$

$$K^2 \int_0^T 2Re[s^*_{ISI}s_{ACI}]dt = K^2 \int_0^T 2Re[s_{ISI}s_{ACI}]dt = AC_R + AD_R + AE_R + BC_R + BD_R + BE_R$$

where

$$AC_R = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{m=0}^{q-1} \varphi_{10} \tag{208}$$

in which

$$\begin{aligned} \varphi_{10} &= \rho^{g+m} b_{0,i} b_{k,0} \frac{1}{j\omega_k} \left[e^{j\omega_k[(1/\beta)(g+(1+i)q)-m/\beta]} - \underbrace{1}_{*} \right] G_6(g,i,m) \\ &\quad + \rho^{g+m} b_{0,i} b^*_{k,0} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k[(1/\beta)(g+(1+i)q)-m/\beta]} - \underbrace{1}_{*} \right] G_6(g,i,m) \end{aligned}$$

where

$$G_6(g,i,m) = \left\{ \begin{array}{ll} 1, & m < g + (1+i)q \\ 0, & \text{otherwise} \end{array} \right\}$$

$*$ *Note:* $e^{\pm j\omega_k(m/\beta - m/\beta)} = e^{j0} = 1$

$$AD_R = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \varphi_{11} \tag{209}$$

in which

$$\varphi_{11} = \rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[e^{j\omega_k[(1/\beta)\min(g+(1+i)q, m+(1+\ell)q)-m/\beta]} - e^{j\omega_k[-m/\beta]} \right]$$

$$+\rho^{g+m} b_{0,i} b_{k,\ell}^{*} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k[(1/\beta)\min(g+(1+i)q, m+(1+\ell)q) - m/\beta]} - e^{-j\omega_k[-m/\beta]} \right]$$

$$AE_R = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \varphi_{12} \tag{210}$$

in which

$$\varphi_{12} = \left[\rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[e^{j\omega_k[(1/\beta)(g+(1+i)q) - m/\beta]} - e^{j\omega_k[\ell q/\beta]} \right] \right.$$

$$\left. +\rho^{g+m} b_{0,i} b_{k,\ell}^{*} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k[(1/\beta)(g+(1+i)q) - m/\beta]} - e^{-j\omega_k[\ell q/\beta]} \right] \right] G_7(g,i,\ell,m)$$

where

$$G_7(g,i,\ell,m) = \left\{ \begin{array}{ll} 1, & m + \ell q < g + (1+i)q \\ 0, & \text{otherwise} \end{array} \right\}$$

$$BC_R = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{m=0}^{q-1} \varphi_{13} \tag{211}$$

in which

$$\begin{aligned} \varphi_{13} \quad = \quad & \rho^{g+m} b_{0,i} b_{k,0} \frac{1}{j\omega_k} \left[e^{j\omega_k[T-(m/\beta)]} - e^{j\omega_k[(1/\beta)\max(g+iq, m) - m/\beta]} \right] \\ & +\rho^{g+m} b_{0,i} b_{k,0}^{*} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k[T-(m/\beta)]} - e^{-j\omega_k[(1/\beta)\max(g+iq, m) - m/\beta]} \right] \end{aligned}$$

$$BD_R = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \varphi_{14} \tag{212}$$

in which

$$\varphi_{14} = \left[\rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[e^{j\omega_k[(1/\beta)(1+\ell)q]} - e^{j\omega_k[(1/\beta)(g+iq) - m/\beta]} \right] \right.$$

$$\left. +\rho^{g+m} b_{0,i} b_{k,\ell}^{*} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k[(1/\beta)(1+\ell)q]} - e^{-j\omega_k[(1/\beta)(g+iq) - m/\beta]} \right] \right] G_8(g,i,\ell,m)$$

where

$$G_8(g,i,\ell,m) = \left\{ \begin{array}{ll} 1, & g+iq < m+(1+\ell)q \\ 0, & \text{otherwise} \end{array} \right\}$$

$$BE_R = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \varphi_{15} \tag{213}$$

in which

$$\begin{aligned} \varphi_{15} \;\; = \;\; & \rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[e^{j\omega_k[T-(m/\beta)]} - e^{j\omega_k[(1/\beta)\max(g+iq,m+\ell q)-m/\beta]} \right] \\ & + \rho^{g+m} b_{0,i} b^*_{k,\ell} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k[T-(m/\beta)]} - e^{-j\omega_k[(1/\beta)\max(g+iq,m+\ell q)-m/\beta]} \right] \end{aligned}$$

We have now computed all terms of

$$\int_0^T |s(t)|^2 dt = K^2 \int_0^T s'(t)s'(t)^* dt \qquad \text{[see Eq. (89)]} \tag{214}$$

Now we recall from Eq. (89) that

$$\begin{aligned} \int_0^T |s(t)|^2 dt = K^2 \int_0^T s'(t)s'(t)^* dt = K^2 \int_0^T |s'(t)|^2 dt \\ = K^2 \int_0^T |s_B|^2 dt + K^2 \int_0^T |s_{ISI}|^2 dt + K^2 \int_0^T |s_{ACI}|^2 dt \\ + K^2 \int_0^T 2Re[s_B s^*_{ISI}]dt + K^2 \int_0^T 2Re[s^*_B s_{ACI}]dt + K^2 \int_0^T [s^*_{ISI} s_{ACI}]dt \end{aligned} \tag{215}$$

We will now extract the answers for the above six integrals from the previous rather lengthy derivation so that we will have the final answer for

$$\int_0^T |s(t)|^2 dt = K^2 \int_0^T |s'(t)|^2 \, dt \tag{216}$$

in one single place. Now, before we consolidate the terms we once again restate

$\max(x_1, x_2)$:	Choose largest of x_1 or x_2, which are both positive. If $x_1 = x_2$, then $\max(x_1, x_2) = x_1 = x_2$.
$\min(x_1, x_2)$:	Choose smallest of x_1 or x_2, which are both positive. If $x_1 = x_2$, then $\min(x_1, x_2) = x_1 = x_2$.

Also recall that $K = \sqrt{P}(1-\rho)$ and $K^2 = P(1-\rho)^2$. Then we have

$$\int_0^T |s(t)|^2 dt = K^2 \int_0^T s'(t)s'(t)^* dt = K^2 \int_0^T |s'(t)|^2 dt \tag{217}$$

which has the following six integrals given by Eq. (215)

$$K^2 \int_0^T |s_B|^2 dt = P(1-\rho)^2 b_{0,0}^2 \sum_{g=0}^{q-1} \sum_{m=0}^{q-1} \rho^{g+m} \left(T - \left(\frac{\max(g,m)}{\beta}\right)\right) \tag{218}$$

For

$$K^2 \int_0^T |s_{ISI}|^2 dt$$

we recall the result of Eq. (117)

$$K^2 \int_0^T s_{ISI} s_{ISI}^* dt = K^2 \int_0^T |s_{ISI}|^2 dt = K^2 \left[\int_0^T \mathcal{A} dt + \int_0^T \mathcal{B} dt \int_0^T \mathcal{C} dt + \int_0^T \mathcal{D} dt\right] \tag{219}$$

$$K^2 \int_0^T \mathcal{A}\, dt = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \varphi_0$$

in which

$$\varphi_0 = \rho^{g+m} b_{0,i} b_{0,r} \left(\frac{\min(g+(1+i)q, m+(1+r)q)}{\beta}\right)$$

$$K^2 \int_0^T \mathcal{B}\, dt + K^2 \int_0^T \mathcal{C}\, dt = 2K^2 \int_0^T \mathcal{B}\, dt =$$

$$2P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} G_1(g,i,m,r)$$

where

$$G_1(g,i,m,r) = \left\{ \begin{array}{ll} \frac{1}{\beta}[(g+(1+i)q) - (m+rq)], & \text{for } m+rq < g+(1+i)q \\ \\ 0, & \text{otherwise} \end{array} \right\}$$

and

$$K^2 \int_0^T \mathcal{D}\, dt \;=\; P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} \left[T - \left(\frac{\max(g+iq, m+rq)}{\beta}\right)\right]$$

We know from Eq. (138) that

$$\begin{aligned}K^2\int_0^T |s_{ACI}|^2 dt &= K^2\int_0^T \mathrm{AA}^* dt + K^2\int_0^T \mathrm{BB}^* dt + K^2\int_0^T \mathrm{CC}^* dt \\ &\quad + K^2\int_0^T 2Re[\mathrm{AC}^*]dt + K^2\int_0^T 2Re[\mathrm{AB}^*]dt \\ &\quad + K^2\int_0^T 2Re[\mathrm{BC}^*]dt \end{aligned} \tag{220}$$

So we have the following six ACI components

$$K^2\int_0^T \mathrm{AA}^* dt = P(1-\rho)^2 \sum_{\substack{k=-M/2\\k\neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2\\n\neq 0}}^{+M/2} \sum_{m=0}^{q-1} \rho^{g+m} \times \varphi_1 \tag{221}$$

where

$$\varphi_1 = \left\{ \begin{array}{ll} b_{k,0}b_{n,0}^* \dfrac{1}{j(\omega_k-\omega_n)} \Big[e^{j[(\omega_k-\omega_n)T-\omega_k(g/\beta)+\omega_n(m/\beta)]} & \\ -e^{j[(\omega_k-\omega_n)(1/\beta)\max(g,m)-\omega_k(g/\beta)+\omega_n(m/\beta)]}\Big], & \text{for } k\neq n \\ \underbrace{b_{k,0}b_{k,0}^*}_{|b_{k,0}|^2} e^{j[(\omega_k/\beta)(m-g)]}\left[T-\frac{1}{\beta}\max(g,m)\right], & \text{for } k=n \end{array} \right\}$$

$$K^2\int_0^T \mathrm{BB}^* dt = P(1-\rho)^2 \sum_{\substack{k=-M/2\\k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \sum_{\substack{n=-M/2\\n\neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \rho^{g+m}\times\varphi_2 \tag{222}$$

where

$$\varphi_2 = \left\{ \begin{array}{l} b_{k,\ell}b_{n,r}^*\left[\dfrac{e^{j[-\omega_k(g/\beta)+\omega_n(m/\beta)]}}{j(\omega_k-\omega_n)}\left(e^{j[(\omega_k-\omega_n)(1/\beta)\min(g+(1+\ell)q,m+(1+r)q)]}-1\right)\right], \\ \hfill \text{for } k\neq n \\ b_{k,\ell}b_{k,r}^*\left(e^{j[(\omega_k/\beta)(m-g)]}\left[\dfrac{1}{\beta}\min(g+(1+\ell)q,m+(1+r)q)\right]\right), \\ \hfill \text{for } k=n \end{array} \right\}$$

$$K^2\int_0^T \mathrm{CC}^* dt = P(1-\rho)^2 \sum_{\substack{k=-M/2\\k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-\ell q}^{-(\ell-1)q-1} \sum_{\substack{n=-M/2\\n\neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} \times \varphi_3 \tag{223}$$

where

$$\varphi_3 = \left\{ \begin{array}{ll} b_{k,\ell} b^*_{n,r} \left[\dfrac{1}{j(\omega_k-\omega_n)} \left[e^{j[(\omega_k-\omega_n)T-\omega_k(g/\beta)+\omega_n(m/\beta)]} \right.\right. & \\ \left.\left. -e^{j[(\omega_k-\omega_n)(1/\beta)\max(g+\ell q, m+rq)-\omega_k(g/\beta)+\omega_n(m/\beta)]} \right] \right], & \text{for } k\neq n \\ b_{k,\ell} b^*_{k,r} e^{j[(\omega_k/\beta)(m-g)]} \left[T-\dfrac{1}{\beta}\max(g+\ell q, m+rq) \right], & \text{for } k=n \end{array} \right\}$$

$$K^2\int_0^T 2Re[\mathrm{AC}^*] = P(1-\rho)^2 \sum_{\substack{k=-M/2\\k\neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2\\n\neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \varphi_4 \tag{224}$$

where

$$\varphi_4 = \left\{ \begin{array}{ll} \rho^{g+m} b_{k,0} b^*_{n,r} \left[\dfrac{1}{j(\omega_k-\omega_n)} \left[e^{j[(\omega_k-\omega_n)T-\omega_k(g/\beta)+\omega_n(m/\beta)]} \right.\right. & \\ \left.\left. -e^{j[(\omega_k-\omega_n)(1/\beta)\max(g, m+rq)-\omega_k(g/\beta)+\omega_n(m/\beta)]} \right] \right] & \\ +\rho^{g+m} b^*_{k,0} b_{n,r} \left[\dfrac{1}{[-j(\omega_k-\omega_n)]} \left[e^{-j[(\omega_k-\omega_n)T-\omega_k(g/\beta)+\omega_n(m/\beta)]} \right.\right. & \\ \left.\left. -e^{-j[(\omega_k-\omega_n)(1/\beta)\max(g, m+rq)-\omega_k(g/\beta)+\omega_n(m/\beta)]} \right] \right], & \text{for } k\neq n \\ \left[b_{k,0} b^*_{k,r} e^{j[(\omega_k/\beta)(m-g)]} + b^*_{k,0} b_{k,r} e^{-j[(\omega_k/\beta)(m-g)]} \right] \rho^{g+m} \times \cdots & \\ \left[T-\dfrac{1}{\beta}\max(g, m+rq) \right], & \text{for } k=n \end{array} \right\}$$

$$K^2 \int_0^T 2Re[\mathrm{AB}^*] = P(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n\neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \varphi_5 \tag{225}$$

where

$$\varphi_5 = \left\{ \begin{array}{ll} \left[\left(\rho^{g+m} b_{k,0} b^*_{n,r} \frac{1}{j(\omega_k - \omega_n)} \left[e^{j[(\omega_k-\omega_n)((m+(1+r)q)/\beta)-\omega_k(g/\beta)+\omega_n(m/\beta)]} \right.\right.\right. & \\ \left.\left. -e^{j[(\omega_n/\beta)(m-g)]} \right] \right) & \\ + \left(\rho^{g+m} b^*_{k,0} b_{n,r} \frac{1}{[-j(\omega_k - \omega_n)]} \left[e^{-j[(\omega_k-\omega_n)((m+(1+r)q)/\beta)-\omega_k(g/\beta)+\omega_n(m/\beta)]} \right.\right. & \\ \left.\left.\left. -e^{-j[(\omega_n/\beta)(m-g)]} \right] \right) \right] G_2(g,m,r), & \text{for } k \neq n \\ \left[b_{k,0} b^*_{k,r} e^{j[(\omega_k/\beta)(m-g)]} + b^*_{k,0} b_{k,r} e^{-j[(\omega_k/\beta)(m-g)]} \right] \rho^{g+m} \times \cdots & \\ \left(\frac{1}{\beta} [(m+(1+r)q) - g] \right) G_2(g,m,r), & \text{for } k = n \end{array} \right\}$$

where

$$G_2(g,m,r) = \left\{ \begin{array}{ll} 1, & \text{for } g < m+(1+r)q \\ 0, & \text{otherwise} \end{array} \right\}$$

$$K^2 \int_0^T 2Re[\mathrm{BC}^*] = P(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \sum_{\substack{n=-M/2 \\ n\neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \varphi_6 \tag{226}$$

where

$$\varphi_6 = \begin{cases} \left(\rho^{g+m} b_{k,\ell} b^*_{n,r} \frac{1}{j(\omega_k - \omega_n)} \left[e^{j[(\omega_k-\omega_n)((g+(1+\ell)q)/\beta)-\omega_k(g/\beta)+\omega_n(m/\beta)]}\right.\right. \\ \left. -e^{j[(\omega_k-\omega_n)((m+rq)/\beta)-\omega_k(g/\beta)+\omega_n(m/\beta)]}\right] \\ +\rho^{g+m} b^*_{k,\ell} b_{n,r} \frac{1}{[-j(\omega_k - \omega_n)]} \left[e^{-j[(\omega_k-\omega_n)((g+(1+\ell)q)/\beta)-\omega_k(g/\beta)+\omega_n(m/\beta)]}\right. \\ \left.\left. -e^{-j[(\omega_k-\omega_n)((m+rq)/\beta)-\omega_k(g/\beta)+\omega_n(m/\beta)]}\right]\right) G_3(g,\ell,m,r), \qquad \text{for } k \neq n \\ \left[b_{k,\ell} b^*_{k,r} e^{j[(\omega_k/\beta)(m-g)]} + b^*_{k,\ell} b_{k,r} e^{-j[(\omega_k/\beta)(m-g)]}\right] \rho^{g+m} \times \cdots \\ \left(\frac{1}{\beta}[(g+(1+\ell)q) - (m+rq)]\right) G_3(g,\ell,m,r), \qquad \text{for } k = n \end{cases}$$

where $G_3(g,\ell,m,r)$ is defined in Eq. (172).

Now, for the last three terms of

$$\int_0^T |s(t)|^2 dt = K^2 \int_0^T s'(t) s'(t)^* dt = K^2 \int_0^T |s'(t)|^2 dt$$

which are

$$K^2 \int_0^T 2Re[s_B s^*_{ISI}] dt\,; \qquad K^2 \int_0^T 2Re[s^*_B s_{ACI}] dt$$

and

$$K^2 \int_0^T 2Re[s^*_{ISI} s_{ACI}] dt$$

$$K^2 \int_0^T 2Re[s_B s^*_{ISI}] dt = K^2 \int_0^T 2[s_B s_{ISI}] dt =$$

$$2P(1-\rho)^2 \sum_{g=0}^{q-1} \sum_{i=-L_0}^{-1} \sum_{m=-(1+i)q+1}^{(-iq)-1} \rho^{g+m} b_{0,0} b_{0,i} G_4(g,i,m)$$

$$+ 2P(1-\rho)^2 \sum_{g=0}^{q-1} \sum_{i=-L_0}^{-1} \sum_{m=-iq}^{-(i-1)q-1} \rho^{g+m} b_{0,0} b_{0,i} \left[T - \frac{1}{\beta}\max(g, m+iq)\right] \tag{227}$$

where

$$G_4(g,i,m) = \left\{ \begin{array}{cc} \frac{1}{\beta}[(m+(1+i)q)-g], & g < m+(1+i)q \\ 0, & \text{otherwise} \end{array} \right\}$$

$$K^2 \int_0^T 2Re[s_B^* s_{ACI}]dt = K^2 \int_0^T 2Re[s_B s_{ACI}]dt = A_{BA} + B_{BA} + C_{BA}$$

$$A_{BA} = P(1-\rho)^2 \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{m=0}^{q-1} \varphi_7$$

in which

$$\begin{aligned} \varphi_7 \; &= \; \rho^{g+m} b_{0,0} b_{k,0} \frac{1}{j\omega_k} \left[e^{j\omega_k (T-(m/\beta))} - e^{j\omega_k ((\max(g,m)-m)/\beta)} \right] \\ &+ \rho^{g+m} b_{0,0} b_{k,0}^* \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k (T-(m/\beta))} - e^{-j\omega_k ((\max(g,m)-m)/\beta)} \right] \end{aligned}$$

$$B_{BA} = P(1-\rho)^2 \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \varphi_8$$

where

$$\begin{aligned} \varphi_8 \; &= \; \rho^{g+m} b_{0,0} b_{k,\ell} \frac{1}{j\omega_k} \left[e^{j\omega_k ((1+\ell)q/\beta)} - e^{j\omega_k ((g-m)/\beta)} \right] G_5(g,\ell,m) \\ &+ \rho^{g+m} b_{0,0} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k ((1+\ell)q/\beta)} - e^{-j\omega_k ((g-m)/\beta)} \right] G_5(g,\ell,m) \end{aligned}$$

where

$$G_5(g,\ell,m) = \left\{ \begin{array}{ll} 1, & g < m+(1+\ell)q \\ 0, & \text{otherwise} \end{array} \right\}$$

$$C_{BA} = P(1-\rho)^2 \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \varphi_9$$

$$
\begin{aligned}
\varphi_9 &= \rho^{g+m} b_{0,0} b_{k,\ell} \frac{1}{j\omega_k} \left[e^{j\omega_k(T-(m/\beta))} - e^{j\omega_k((\max(g,m+\ell q)-m)/\beta)} \right] \\
&+ \rho^{g+m} b_{0,0} b^*_{k,\ell} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k(T-(m/\beta))} - e^{-j\omega_k((\max(g,m+\ell q)-m)/\beta)} \right] \qquad (228)
\end{aligned}
$$

$$
\begin{aligned}
K^2 \int_0^T 2Re[s^*_{ISI} s_{ACI}]dt &= K^2 \int_0^T 2Re[s_{ISI} s_{ACI}]dt \\
&= AC_R + AD_R AE_R + BC_R + BD_R + BE_R
\end{aligned}
$$

$$
AC_R = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{m=0}^{q-1} \varphi_{10} \qquad (229)
$$

where

$$
\begin{aligned}
\varphi_{10} &= \rho^{g+m} b_{0,i} b_{k,0} \frac{1}{j\omega_k} \left[e^{j\omega_k[(1/\beta)(g+(1+i)q)-m/\beta]} - \underbrace{1}_{*} \right] G_6(g,i,m) \\
&+ \rho^{g+m} b_{0,i} b^*_{k,0} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k[(1/\beta)(g+(1+i)q)-m/\beta]} - \underbrace{1}_{*} \right] G_6(g,i,m)
\end{aligned}
$$

where

$$
G_6(g,i,m) = \left\{ \begin{array}{ll} 1, & m < g + (1+i)q \\ 0, & \text{otherwise} \end{array} \right\}
$$

$*$ *Note:* $e^{\pm j\omega_k(m/\beta - m/\beta)} = e^{j0} = 1$

$$
AD_R = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \varphi_{11} \qquad (230)
$$

in which

$$
\begin{aligned}
\varphi_{11} &= \rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[e^{j\omega_k[(1/\beta)\min(g+(1+i)q, m+(1+\ell)q)-m/\beta]} - e^{j\omega_k[-m/\beta]} \right] \\
&+ \rho^{g+m} b_{0,i} b^*_{k,\ell} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k[(1/\beta)\min(g+(1+i)q, m+(1+\ell)q)-m/\beta]} - e^{-j\omega_k[-m/\beta]} \right]
\end{aligned}
$$

$$AE_R = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \varphi_{12} \tag{231}$$

in which

$$\varphi_{12} = \left[\rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[e^{j\omega_k[(1/\beta)(g+(1+i)q)-m/\beta]} - e^{j\omega_k[\ell q/\beta]}\right]\right.$$
$$\left. +\rho^{g+m} b_{0,i} b^*_{k,\ell} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k[(1/\beta)(g+(1+i)q)-m/\beta]} - e^{-j\omega_k[\ell q/\beta]}\right]\right] G_7(g,i,\ell,m)$$

where

$$G_7(g,i,\ell,m) = \left\{ \begin{array}{ll} 1, & m+\ell q < g+(1+i)q \\ 0, & \text{otherwise} \end{array} \right\}$$

$$BC_R = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{m=0}^{q-1} \varphi_{13} \tag{232}$$

in which

$$\begin{aligned} \varphi_{13} &= \rho^{g+m} b_{0,i} b_{k,0} \frac{1}{j\omega_k} \left[e^{j\omega_k[T-(m/\beta)]} - e^{j\omega_k[(1/\beta)\max(g+iq,m)-m/\beta]}\right] \\ &+\rho^{g+m} b_{0,i} b^*_{k,0} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k[T-(m/\beta)]} - e^{-j\omega_k[(1/\beta)\max(g+iq,m)-m/\beta]}\right] \end{aligned}$$

$$BD_R = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \varphi_{14} \tag{233}$$

in which

$$\begin{aligned} \varphi_{14} &= \left[\rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[e^{j\omega_k[(1/\beta)(1+\ell)q]} - e^{j\omega_k[(1/\beta)(g+iq)-m/\beta]}\right]\right. \\ &\left. +\rho^{g+m} b_{0,i} b^*_{k,\ell} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k[(1/\beta)(1+\ell)q]} - e^{-j\omega_k[(1/\beta)(g+iq)-m/\beta]}\right]\right] G_8(g,i,\ell,m) \end{aligned}$$

where

$$G_8(g,i,\ell,m) = \left\{ \begin{array}{ll} 1, & g+iq < m+(1+\ell)q \\ 0, & \text{otherwise} \end{array} \right\}$$

$$BE_R = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \varphi_{15} \tag{234}$$

in which

$$\begin{aligned} \varphi_{15} &= \rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[e^{j\omega_k[T-(m/\beta)]} - e^{j\omega_k[(1/\beta)\max(g+iq,m+\ell q)-m/\beta]} \right] \\ &+ \rho^{g+m} b_{0,i} b^*_{k,\ell} \frac{1}{[-j\omega_k]} \left[e^{-j\omega_k[T-(m/\beta)]} - e^{-j\omega_k[(1/\beta)\max(g+iq,m+\ell q)-m/\beta]} \right] \end{aligned}$$

We have now laid down all terms of

$$\int_0^T |s(t)|^2 dt = K^2 \int_0^T s'(t)s'(t)^* dt = K^2 \int_0^T |s'(t)|^2 dt$$

This will be used to compute the detection statistic X where

$$X = \mathcal{R} \int_0^T |s(t)|^2 dt$$

where $\mathcal{R}$ is the responsivity of the photodetector (A/W). This will be used to compute probabilities of bit error for the dense WDM system.

APPENDIX B

LIMITED CASE OF THE COMPLETE MODEL (10 PROGRAMMABLE TERMS) AND RESULTING PROBABILITY OF BIT ERROR EQUATIONS

In Appendix A we derived the complete expression for $\int_0^T |s(t)|^2 dt$. Referring to the OOK receiver structure in Fig. 1, we note that the deterministic signal component of the random variable appearing at the output of the integrator is

$$X = \mathcal{R} \int_0^T |s(t)|^2 dt \tag{235}$$

where $\mathcal{R}$ is the responsivity of the photodetector (A/W) and $s(t)$ is the complex baseband output of the Fabry-Perot filter. Note that the photodetector detects $s(t)$ and produces an output current $\mathcal{R}|s(t)|^2$. We will call the integrator output the decision variable Y where

$$Y = X + N \tag{236}$$

We note that N is a Gaussian random variable with mean zero and variance $N_0 T$

$$N \sim \mathbf{N}(0, N_0 T) \tag{237a}$$

and

$$N = \int_0^T n(t) dt \tag{237b}$$

where $n(t)$ is the postdetection additive white Gaussian noise and N_0 (A^2/Hz) is the two-sided current spectral density of $n(t)$.

Now we will set up a fairly limited but realistic case for which we will program the appropriate terms derived in Appendix A and generate the probability of bit error. Letting

$$\begin{aligned} L_0 &= 1 \qquad L = 0 \\ \phi_k &= 0 \qquad \omega_k = \frac{2\pi k I}{T} \end{aligned}$$

$\omega_k = 2\pi kI/T$ is a special case of the radian frequency spacing between Channel 0 and Channel k. Recall again that I is the normalized channel spacing integer ($I > 0$), and T is the data bit period (s). $L = 0$ and $\phi_k = 0$ means that we will only model and deal with adjacent channel interference (ACI) from the bits in the adjacent channels which occur during the detection window $0 \leq t \leq T$. Recall that the adjacent channels are symmetric in frequency around Channel 0, the channel of interest. $L_0 = 1$ means that we will model the effects of a single ISI bit trailing the detected bit of interest $b_{0,0}$ in Channel 0. $\phi_k = 0$ means that the phase offset between Channel 0 and Channel k is 0, or that all adjacent channels are bit synchronous with with Channel 0.

Then, since $\phi_k = 0$, $L = 0$, and $b_{k,\ell} \in \{0, e^{j\phi_k}\}$, we let

$$b_{-M/2,0} = b_{-M/2+1,0} = \cdots = b_{-1,0} = b^- \quad \text{and} \quad b^- \in \{0, 1\} \tag{238}$$

b^- is the left adjacent channel bit pattern and all $M/2$ of the left channel bits will be 1 or 0 simultaneously during detection window $0 \leq t \leq T$. And again, since $\phi_k = 0$, $L = 0$, and $b_{k,\ell} \in \{0, e^{\phi_k}\}$, we let

$$b_{M/2,0} = b_{M/2-1,0} = \cdots = b_{1,0} = b^+ \quad \text{and} \quad b^+ \in \{0, 1\} \tag{239}$$

b^+ is the right adjacent channel bit pattern and all $M/2$ of the right channel bits will be 1 or 0 simultaneously during the detection window $0 \leq t \leq T$. Also, we know $L_0 = 1$, and $b_{0,i} \in \{0, 1\}$, so using Eq. (1) or Eq. (41) we have

$$b_{0,-1} \in \{0, 1\} \quad \text{and} \quad b_{0,0} \in \{0, 1\} \tag{240}$$

We define an ACI/ISI bit pattern

$$\psi_p = \{b^-, b^+, b_{0,-1}\} \tag{241}$$

Note that b^-, b^+, and $b_{0,-1}$ are 0 or 1 with probability 1/2 yielding eight possible values of ψ_p.

Now let use define an ACI/ISI bit pattern set ψ

$$\psi = \{\psi_1, \psi_2, \cdots, \psi_8\} \tag{242}$$

We can denote each individual element in the set ψ as ψ_p where $p = 1, 2, 3, \ldots, 8$ as there are eight possible values of ψ_p, and

$$\psi = \{\psi_p\} \tag{243}$$

where $p = 1, \cdots, 8$. Now we define

$$X_0(\psi_p) = X(\psi_p, b_{0,0} = 0) \quad \text{or} \quad X \text{ evaluated at the current value of } \psi_p \text{ with } b_{0,0} = 0 \tag{244}$$

$$X_1(\psi_p) = X(\psi_p, b_{0,0} = 1) \quad \text{or} \quad X \text{ evaluated at the current value of } \psi_p \text{ with } b_{0,0} = 1 \tag{245}$$

It can be shown that the conditional probability of error for the dense WDM system given the ACI/ISI bit pattern ψ_p is [2, 4]

$$P\left(\text{error}|\psi_p\right) = \frac{1}{2}Q\left(\frac{X_1(\psi_p) - V_T}{\sqrt{N_0 T}}\right) + \frac{1}{2}Q\left(\frac{V_T - X_0(\psi_p)}{\sqrt{N_0 T}}\right) \tag{246}$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy \tag{247}$$

There are eight possible members of the set ψ

$$\psi = \left\{ \begin{array}{llll} \psi_1 = 0 \;\; 0 \;\; 0 & & \psi_5 = 1 \;\; 0 \;\; 0 \\ \psi_2 = 0 \;\; 0 \;\; 1 & & \psi_6 = 1 \;\; 0 \;\; 1 \\ \psi_3 = 0 \;\; 1 \;\; 0 & & \psi_7 = 1 \;\; 1 \;\; 0 \\ \psi_4 = 0 \;\; 1 \;\; 1 & & \psi_8 = 1 \;\; 1 \;\; 1 \end{array} \right\} \tag{248}$$

V_T is the detection threshold where

$$V_T = \frac{X_0 \max + X_1 \min}{2} \tag{249}$$

and

$$X_0\max = \max_{\{\psi_p\}} \left(X_0(\psi_p)\right) \tag{250}$$

i.e., to find $X_0\max$, compute all eight values of X_0 [see Eq. (244)] and then choose the maximum, and

$$X_1\min = \min_{\{\psi_p\}} \left(X_1(\psi_p)\right) \tag{251}$$

i.e., to find $X_1\min$, compute all eight values of X_1 [see Eq. (245)] and then choose the minimum. Then, by the law of total probability, the probability of bit error is

$$P_b = P\left(\text{error}|\psi_1\right) P(\psi_1) + P\left(\text{error}|\psi_2\right) P(\psi_2) + \cdots + P\left(\text{error}|\psi_8\right) P(\psi_8) \tag{252}$$

Assuming all bit patterns are equiprobable

$$P(\psi_1) = P(\psi_2) = \cdots = P(\psi_8) = \frac{1}{8} \tag{253}$$

Then we can easily see that

$$P_b = \frac{1}{8}\sum_{p=1}^{8} P\left(\text{error}|\psi_p\right) \tag{254}$$

Again, we note that

$$X = \mathcal{R}\int_0^T |s(t)|^2 dt \tag{255}$$

where $\int_0^T |s(t)|^2 dt$ was computed in Appendix A and consisted of 21 terms ("clusters of summations") [see Eqs. (218)–(234)]. Given our parameters $L_0 = 1$, $L = 0$, $\phi_k = 0$, and $\omega_k = 2\pi kI/T$, 11 of the 21 terms drop out and we are left to compute in order ten to generate the probability of bit error. If we name these ten terms ("clusters of summations") $\text{SUM}_1, \text{SUM}_2, \cdots$, and SUM_{10}, then for $L_0 = 1$, $L = 0$, $\phi_k = 0$, and $\omega_k = 2\pi kI/T$

$$\int_0^T |s(t)|^2 dt = \text{SUM}_1 + \text{SUM}_2 + \cdots + \text{SUM}_{10} \tag{256}$$

and using Eq. (255)

$$X = \mathcal{R}(\text{SUM}_1 + \text{SUM}_2 + \cdots + \text{SUM}_{10}) \tag{257}$$

Now PT may be factored out of each sum. So we define

$$\text{SUM}_j = PT(\text{QUOT}_j) \qquad j = 1, \cdots, 10 \tag{258}$$

So

$$X = \mathcal{R}\,PT(\text{QUOT}_1) + \mathcal{R}\,PT(\text{QUOT}_2) + \cdots + \mathcal{R}\,PT(\text{QUOT}_{10}) \tag{259}$$

and

$$X = \mathcal{R}\,PT[\text{QUOT}_1 + \text{QUOT}_2 + \cdots + \text{QUOT}_{10}] \tag{260}$$

Also, letting $\text{QUOT}_1 + \text{QUOT}_2 + \cdots + \text{QUOT}_{10} = \text{QUOTSUM}$, we also see

$$X = \mathcal{R}\,PT[\text{QUOTSUM}] \tag{261}$$

We can also say that, in general, $\text{QUOT}_1, \text{QUOT}_2, \cdots, \text{QUOT}_{10}$, and QUOTSUM are dependent functions of ψ_p and $b_{0,0}$. Thus, we may write in general

$$\text{QUOT}_1(\psi_p, b_{0,0}), \text{QUOT}_2(\psi_p, b_{0,0}), \cdots, \text{QUOT}_{10}(\psi_p, b_{0,0})$$

and

$$\text{QUOTSUM}(\psi_p, b_{0,0})$$

Now we can see that

$$X_1(\psi_p) = \mathcal{R}\,PT[\text{QUOTSUM}(\psi_p, b_{0,0} = 1)] \tag{262}$$

$$X_0(\psi_p) = \mathcal{R}\,PT[\text{QUOTSUM}(\psi_p, b_{0,0} = 0)] \tag{263}$$

Recalling that

$$V_T = \frac{X_0 \max + X_1 \min}{2}$$

we let

$$\psi_{p\,\max} = \text{Value of } \psi_p \text{ which causes maximum value of of } X_0 \text{ (or maximum value of} X \text{ with } b_{0,0} = 0). \tag{264}$$

and

$$\psi_{p\ \min} = \text{Value of } \psi_p \text{ which causes minimum value of of } X_1 \text{ (or minimum value of} X \text{ with } b_{0,0} = 1). \tag{265}$$

Then using Eqs. (249), (262)–(265)

$$V_T = \frac{\mathcal{R}PT[\text{QUOTSUM}(\psi_{p\ \max}, b_{0,0} = 0) + \text{QUOTSUM}(\psi_{p\ \min}, b_{0,0} = 1)]}{2} \tag{266}$$

Now using Eqs. (246), (262), (263), and (266)

$P(\text{error}/\psi_p) =$

$$\frac{1}{2}Q\left(\left(\frac{\mathcal{R}PT}{\sqrt{N_0T}}\right)\left([\text{QUOTSUM}(\psi_p, b_{0,0} = 1)] - \left[\frac{\text{QUOTSUM}(\psi_{p\ \max}, b_{0,0} = 0) + \text{QUOTSUM}(\psi_{p\ \min}, b_{0,0} = 1)}{2}\right]\right)\right)$$
$$+\frac{1}{2}Q\left(\left(\frac{\mathcal{R}PT}{\sqrt{N_0T}}\right)\left(\left[\frac{\text{QUOTSUM}(\psi_{p\ \max}, b_{0,0} = 0) + \text{QUOTSUM}(\psi_{p\ \min}, b_{0,0} = 1)}{2}\right] - [\text{QUOTSUM}(\psi_p, b_{0,0} = 0)]\right)\right) \tag{267}$$

Thus, we see that $P(\text{error}|\psi_p)$ is directly related to the signal to noise ratio which we will call Z.

$$Z = \frac{\mathcal{R}PT}{\sqrt{N_0T}} = \mathcal{R}P\sqrt{\frac{T}{N_0}} \tag{268}$$

Thus, when we compute a probability of the bit error graph, we will choose a suitable range of values for Z, a value of free spectral range-bit period product βT, and several values of the number of adjacent channels M. For each of these values of M, we will compute a probability of bit error trace. To compute a point on a trace, choose a value of Z, compute all eight values of $P(\text{error}|\psi_p)$, sum all eight values of $P(\text{error}/\psi_p)$, and divide by 8 [see Eqs. (248), (254), and (267)]. The point is then plotted.

Recall that

$$q = \frac{T}{\frac{1}{\beta}} \tag{269a}$$

then

$$q = \beta T \tag{269b}$$

We will use this and the fact that [see Eq. (259)]

$$X = \underbrace{\mathcal{R}\,PT(\text{QUOT}_1)}_{\text{Term \#1}} + \underbrace{\mathcal{R}\,PT(\text{QUOT}_2)}_{\text{Term \#2}} + \cdots + \underbrace{\mathcal{R}\,PT(\text{QUOT}_{10})}_{\text{Term \#10}} \tag{270}$$

to compute the probability of bit error graphs. We also use Eqs. (256), (257), (258), (260), (261), (262), (263), (264), (265), (266), and (267) in this endeavor.

We now proceed to write out the ten terms of X, which we will use to compute the probability of bit error graphs. Before we begin we recall that max (x_1, x_2) means choose the largest of x_1 or x_2. If $x_2 = x_2$, then max $(x_1, x_2) = x_1 = x_2$. Also recall that $\min(x_1, x_2)$ means choose the smallest of x_1 or x_2. If $x_1 = x_2$, then min $(x_1, x_2) = x_1 = x_2$. The first term to be programmed will be Eq. (218), Appendix A. Factoring out a T we get

$$\text{SUM}_1 = PT(1-\rho)^2 b_{0,0}^2 \sum_{g=0}^{q-1} \sum_{m=0}^{q-1} \rho^{g+m} \left(1 - \left(\frac{\max(g,m)}{\beta T}\right)\right) \tag{271}$$

Using $q = \beta T$ and multiplying by $\mathcal{R}$ we get

$$\text{Term \#1} = \mathcal{R}\,PT(1-\rho)^2 b_{0,0}^2 \sum_{g=0}^{\beta T-1} \sum_{m=0}^{\beta T-1} \rho^{g+m} \left(1 - \left(\frac{\max(g,m)}{\beta T}\right)\right) \tag{272}$$

With $L_0 = 1$ and $q = \beta T$, the first term ("cluster of summations") in Eq. (219), Appendix A, becomes

$$P(1-\rho)^2 \sum_{g=1}^{\beta T-1} \sum_{m=1}^{\beta T-1} (b_{0,-1})^2 \left(\frac{1}{\beta} \min(g,m)\right) \tag{273}$$

Factoring out a T and multiplying by $\mathcal{R}$ we get

$$\text{Term \#2} = \mathcal{R}\,PT(1-\rho)^2 (b_{0,-1})^2 \sum_{g=1}^{\beta T-1} \sum_{m=1}^{\beta T-1} \rho^{g+m} \frac{1}{\beta T} \min(g,m) \tag{274}$$

With $L_0 = 1$ and $q = \beta T$, the second term ("cluster of summations") in Eq. (219), Appendix A, becomes

$$2P(1-\rho)^2 \sum_{g=1}^{\beta T-1} \sum_{m=\beta T}^{2\beta T-1} \rho^{g+m}(b_{0,-1})^2 G_1(g, i=-1, m, r=-1) \tag{275}$$

where

$$G_1(g, i=-1, m, r=-1) = \left\{ \begin{array}{ll} \frac{1}{\beta}[g-(m-\beta T)], & \text{for } m-\beta T < g \\ \\ 0, & \text{otherwise} \end{array} \right\}$$

Factoring a T and multiplying by $\mathcal{R}$, we arrive at the expression for Term #3

$$\text{Term \#3} = \mathcal{R}\, PT \times 2(1-\rho)^2(b_{0,-1})^2 \sum_{g=1}^{\beta T-1} \sum_{m=\beta T}^{2\beta T-1} \rho^{g+m} \left[\frac{1}{T} \times G_1(g, i=-1, m, r=-1)\right] \tag{276}$$

With $L_0 = 1$ and $q = \beta T$, the third term ("cluster of summations") in Eq. (219), Appendix A, yields Term #4, after factoring a T and multiplying by $\mathcal{R}$.

$$\text{Term \#4} = \mathcal{R}\, PT(1-\rho)^2(b_{0,-1})^2 \sum_{g=\beta T}^{2\beta T-1} \sum_{m=\beta T}^{2\beta T-1} \rho^{g+m} \left[1 - \frac{1}{\beta T}\max(g-\beta T, m-\beta T)\right] \tag{277}$$

The next applicable term is Eq. (221) of Appendix A. Our given parameters include $\phi_k = \phi_n = 0$. We can then say by recalling that $b_{k,\ell} \in \{0, e^{j\phi_k}\}$

$$b^*_{n,0} = b_{n,0} \quad \text{and} \quad b^*_{k,0} = b_{k,0} \tag{278}$$

We will also use the facts

$$\omega_k = \frac{2\pi k I}{T} \quad \text{and} \quad \omega_n = \frac{2\pi n I}{T} \tag{279}$$

where I is a positive integer called the normalized channel spacing integer. Now, by slightly reworking the $\omega_k \neq \omega_n$ expression by substituting the results given in Eqs. (278) and (279) into the appropriate places, we get

$$\begin{aligned} \varphi_1 \quad = \quad & b_{k,0}b_{n,0}\frac{T}{j[2\pi I(k-n)]}\Big[e^{j[2\pi I(k-n)-(2\pi kI(g)/\beta T)+(2\pi nI(m)/\beta T)]} \\ & -e^{j[(2\pi I/\beta T)(k-n)\max(g,m)-(2\pi kI(g)/\beta T)+(2\pi nI(m)/\beta T)]}\Big], \qquad \text{for } k \neq n \end{aligned} \tag{280}$$

Note: The $2\pi I(k-n)$ term in the first phasor of Eq. (280) contributes integer multiples of 2π to the phasor, and so it is neglected in the next rewrite of the term. Then, by factoring the common factor $e^{j(2\pi I/\beta T)[(nm)-(kg)]}$, we arrive at

$$\varphi_1 = \frac{T b_{k,0} b_{n,0} e^{j(2\pi I/\beta T)[(nm)-(kg)]}}{j[2\pi I(k-n)]}\left[1 - e^{j(2\pi I/\beta T)(k-n)\max(g,m)}\right], \quad \text{for } k \neq n \tag{281}$$

Finally, after we factor out a T from both the $\omega_k \neq \omega_n$ and $\omega_k = \omega_n$ arguments to the summations, rearrange the summations for more logical programming, and use the fact that $q = \beta T$, we arrive at the final form for Term #5.

$$\text{Term } \#5 = \mathcal{R}\,PT(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{\substack{n=-M/2 \\ n\neq 0}}^{+M/2} (b_{k,0})(b_{n,0}) \sum_{g=0}^{\beta T-1} \sum_{m=0}^{\beta T-1} \rho^{g+m} \times \varphi_1 \tag{282}$$

where

$$\varphi_1 = \left\{ \begin{array}{ll} \dfrac{e^{j(2\pi I/\beta T)[(nm)-(kg)]}}{j[2\pi I(k-n)]}\left[1 - e^{j(2\pi I/\beta T)(k-n)\max(g,m)}\right] & \text{for } k \neq n \\ e^{j[(2\pi k I/\beta T)(m-g)]}\left[1 - \dfrac{\max(g,m)}{\beta T}\right], & \text{for } k = n \end{array} \right\}$$

The next applicable term is the first term ("cluster of summations") in Eq. (227), Appendix A. Utilizing $L_0 = 1$, factoring a T, multiplying by $\mathcal{R}$, and noting that $q = \beta T$ yields

$$\text{Term } \#6 = \mathcal{R}\,PT \times 2(1-\rho)^2(b_{0,0})(b_{0,-1}) \sum_{g=0}^{\beta T-1} \sum_{m=1}^{\beta T-1} \rho^{g+m} \frac{G_4(g, i=-1, m)}{T} \tag{283}$$

where

$$\frac{G_4(g, i=-1, m)}{T} = \left\{ \begin{array}{ll} \dfrac{1}{\beta T}[m-g], & \text{for } g < m \\ 0, & \text{otherwise} \end{array} \right\}$$

Looking at the second term ("cluster of summations") in Eq. (227), Appendix A, and after utilizing $L_0 = 1$, factoring a T, multiplying by $\mathcal{R}$, and substituting

$q = \beta T$, we have

$$\text{Term } \#7 = \mathcal{R}\, PT \times 2(1-\rho)^2 (b_{0,0})(b_{0,-1}) \sum_{g=0}^{\beta T-1} \sum_{m=\beta T}^{2\beta T-1} \rho^{g+m} \left[1 - \frac{1}{\beta T}\max(g, m-\beta T)\right] \tag{284}$$

The next applicable term is the first term ("cluster of summations") in Eq. (228), Appendix A. Since $\phi_k = 0$

$$b^*_{k,0} = b_{k,0} \tag{285}$$

We can then work with the term inside of the summations

$$\begin{aligned} \varphi_7 &= \rho^{g+m} b_{0,0} b_{k,0} \frac{1}{j\omega_k}\left[e^{j\omega_k(T-(m/\beta))} - e^{j\omega_k((\max(g,m)-m)/\beta)}\right] \\ &\quad + \rho^{g+m} b_{0,0} b^*_{k,0} \frac{1}{[-j\omega_k]}\left[e^{-j\omega_k(T-(m/\beta))} - e^{-j\omega_k((\max(g,m)-m)/\beta)}\right] \end{aligned} \tag{286}$$

Then, since $b^*_{k,0} = b_{k,0}$ and after some rearranging, we have

$$\begin{aligned} \varphi_7 &= \frac{\rho^{g+m} b_{0,0} b_{k,0}}{\omega_k} \times \cdots \\ &\left[\frac{e^{j\omega_k(T-(m/\beta))} - e^{-j\omega_k(T-(m/\beta))}}{j} \underbrace{\frac{-e^{j\omega_k((\max(g,m)-m)/\beta)} + e^{-j\omega_k((\max(g,m)-m)/\beta)}}{j}}_{\Lambda}\right] \end{aligned} \tag{287}$$

Then factoring a -1 from Λ yields

$$\Lambda = -\left(\frac{e^{j\omega_k((\max(g,m)-m)/\beta)} - e^{-j\omega_k((\max(g,m)-m)/\beta)}}{j}\right) \tag{288}$$

Euler's Relationship defines

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j} \tag{289}$$

Applying this relationship yields

$$\varphi_7 = \frac{2\rho^{g+m} b_{0,0} b_{k,0}}{\omega_k}\left[\sin\left(\omega_k\left(T - \frac{m}{\beta}\right)\right) - \sin\left(\omega_k\left(\frac{\max(g,m)-m}{\beta}\right)\right)\right] \tag{290}$$

and since $\omega_k = 2\pi kI/T$

$$\varphi_7 = \cancel{2}\frac{\rho^{g+m} b_{0,0} b_{k,0}}{\left(\frac{\cancel{2}\pi kI}{T}\right)} \left[\sin\left[2\pi kI\left(1 - \frac{m}{\beta T}\right)\right] - \sin\left[\frac{2\pi kI}{\beta T}(\max(g,m) - m)\right]\right] \tag{291}$$

After factoring a T, multiplying by $\mathcal{R}$, utilizing $q = \beta T$, and rearranging the summations for more logical programming, we arrive at the expression for Term #8.

$$\text{Term } \#8 = \mathcal{R}\, PT(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{g=0}^{\beta T-1} \sum_{m=0}^{\beta T-1} \varphi_7$$

where

$$\varphi_7 = \frac{\rho^{g+m} b_{0,0} b_{k,0}}{\pi kI} \left[\sin\left[2\pi kI\left(1 - \frac{m}{\beta T}\right)\right] - \sin\left[\frac{2\pi kI}{\beta T}(\max(g,m) - m)\right]\right] \tag{292}$$

The next applicable term ("cluster of summations") is Eq. (229) of Appendix A. Since $\phi_k = 0$, we know $b_{k,0} = b^*_{k,0}$. Using $L_0 = 1$, we can rework the argument of the summations. Making the appropriate parameter substitutions yields

$$\begin{aligned} \varphi_{10} &= \rho^{g+m} b_{0,-1} b_{k,0} \frac{1}{j\omega_k}\left[e^{j\omega_k[(g-m)/\beta]} - 1\right] G_6(g, i=-1, m) \\ &\quad + \rho^{g+m} b_{0,-1} b_{k,0} \frac{1}{[-j\omega_k]}\left[e^{-j\omega_k[(g-m)/\beta]} - 1\right] G_6(g, i=-1, m) \end{aligned} \tag{293}$$

Further symplifying yields

$$\varphi_{10} = \frac{\rho^{g+m}(b_{0,-1})(b_{k,0})}{\omega_k} G_6(g, i=-1, m) \left[\underbrace{\frac{e^{j\omega_k[(g-m)/\beta]} - e^{-j\omega_k[(g-m)/\beta]}}{j}}_{2\sin[\omega_k[(g-m)/\beta]]}\right] \tag{294}$$

Using $\omega_k = 2\pi kI/T$ yields

$$\varphi_{10} = \frac{T\rho^{g+m}(b_{0,-1})(b_{k,0})}{\pi kI} G_6(g, i=-1, m) \sin\left[\frac{2\pi kI}{\beta T}(g-m)\right] \tag{295}$$

Substituting φ_{10} inside of the summations, multiplying by $\mathcal{R}$, factoring T, using $q = \beta T$, and rearranging the summations for logical programming yields the following for Term #9.

$$\text{Term } \#9 = \mathcal{R}\, PT(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{g=1}^{\beta T-1} \sum_{m=0}^{\beta T-1} \varphi_{10} \tag{296}$$

where

$$\varphi_{10} = \frac{\rho^{g+m}(b_{0,-1})(b_{k,0})}{\pi k I} G_6(g, i=-1, m) \sin\left[\frac{2\pi k I}{\beta T}(g-m)\right]$$

amd

$$G_6(g, i=-1, m) = \left\{ \begin{array}{ll} 1, & \text{for } m < g \\ 0, & \text{otherwise} \end{array} \right\}$$

Look at the last applicable term ("cluster of summations"), Eq. (232), Appendix A. Again we take advantage of the fact that $\phi_k = 0$, which means $b_{k,0} = b^*_{k,0}$. Making the appropriate parameter substitutions yields

$$\varphi_{13} = \frac{\rho^{g+m}(b_{0,-1})(b_{k,0})}{\omega_k}\frac{1}{j}\left[e^{j\omega_k[T-(m/\beta)]} - e^{-j\omega_k[T-(m/\beta)]}\right.$$

$$\left. - \left[e^{j\omega_k[(1/\beta)\max(g-\beta T,m)-m/\beta]} - e^{-j\omega_k[(1/\beta)\max(g-\beta T,m)-m/\beta]}\right]\right] \tag{297}$$

Using Euler's Relationship we get

$$\varphi_{13} = \frac{T\rho^{g+m}(b_{0,-1})(b_{k,0})}{\pi k I}\left[\sin\left[2\pi k I\left(1-\frac{m}{\beta T}\right)\right] - \sin\left[\frac{2\pi k I}{\beta T}(\max(g-\beta T,m)-m)\right]\right] \tag{298}$$

Substituting φ_{13} back inside of the summations, factoring a T, multiplying by $\mathcal{R}$, utilizing $q = \beta T$, and rearranging the summations for logical programming yields

$$\text{Term } \#10 = \mathcal{R}\, PT(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{+M/2} \sum_{g=\beta T}^{2\beta T-1} \sum_{m=0}^{\beta T-1} \varphi_{13} \tag{299}$$

where

$$\varphi_{13} = \frac{\rho^{g+m}(b_{0,-1})(b_{k,0})}{\pi k I}\left[\sin\left[2\pi k I\left(1-\frac{m}{\beta T}\right)\right] - \sin\left[\frac{2\pi k I}{\beta T}(\max(g-\beta T,m)-m)\right]\right]$$

For the given parameters $L_0 = 1$, $L = 0$, $\phi_k = 0$, and $\omega_k = 2\pi kI/T$, we have now derived all ten programmable terms of X where

$$X = \underbrace{\mathcal{R}\,PT(\mathrm{QUOT}_1)}_{\text{Term \#1}} + \underbrace{\mathcal{R}\,PT(\mathrm{QUOT}_2)}_{\text{Term \#2}} + \cdots + \underbrace{\mathcal{R}\,PT(\mathrm{QUOT}_{10})}_{\text{Term \#10}} \tag{300}$$

We will use these ten programmable terms to compute probability of bit error for the dense WDM system.

APPENDIX C

PROGRAMMING STRATEGY AND COMPUTER PROGRAMS FOR GENERATION OF PROBABILITY OF BIT ERROR GRAPHS

In this appendix, we present the computer programs written to generate the probability of bit error graphs for the system under consideration. We attempt to generate four graphs, one for each value of βT considered. We reiterate that β is the free spectral range of the Fabry-Perot filter (Hz) and that T is the data bit period (s). βT is called the free spectral range-bit period product. The four values of βT used are

$$\beta T = [500 \quad 1000 \quad 1500 \quad 2000]$$

In each of the four graphs there will be five traces as we will present probability bit error for four values of the normalized channel spacing integer I, or equivalently the number of adjacent channels M, along with a probability of bit error trace for single channel (SC) operation without Fabry-Perot (FP) filtering or SC operation with FP filtering and without ISI or ACI. We will show the relationship between I and the number of adjacent channels M later. Now, however, we present the four values of I corresponding to each value of βT:

For $\beta T = 500$

$$I = [4 \quad 5 \quad 8 \quad 20]$$

For $\beta T = 1000$

$$I = [5 \quad 6 \quad 9 \quad 20]$$

For $\beta T = 1500$

$$I = [7 \quad 9 \quad 12 \quad 20]$$

For $\beta T = 2000$

$$I = [8 \quad 9 \quad 12 \quad 20]$$

Without a lengthy discourse, we present the mathematical relationship between the normalized channel spacing integer I and the number of adjacent channels M

$$M = \frac{\beta}{\Delta f} - 1 \tag{301}$$

For the special case $f_k = k\Delta f = kI/T$ we can easily see that $\Delta f = I/T$. Then

$$M = \frac{\beta T}{I} - 1 \tag{302}$$

For the values of I above M will not always be an even integer. Thus, an algorithm to consistently arrive at an even integer value of M which is less than or equal to the true mathematical value presented in Eq. (302) was devised. To get the number of adjacent channels, we perform the following operation

$$\frac{\beta T}{I} = Q + R \tag{303}$$

where Q is the integer quotient of the division operation and R is the remainder. To arrive at M we

1. Subtract 1 from Q if Q is an odd integer and $R = 0$.
2. Subtract 2 from Q if Q is an even integer and $R = 0$.
3. Subtract 1 from Q if Q is an odd integer and $R \neq 0$.
4. Subtract 2 from Q if Q is an even integer and $R \neq 0$.

Using these rules, we obtain at the four values of M for each value of βT.

For $\beta T = 500$

$$M = [\underbrace{124}_{I=4} \quad \underbrace{98}_{I=5} \quad \underbrace{60}_{I=8} \quad \underbrace{24}_{I=20}]$$

For $\beta T = 1000$

$$M = [\underbrace{198}_{I=5} \quad \underbrace{164}_{I=6} \quad \underbrace{110}_{I=9} \quad \underbrace{48}_{I=20}]$$

For $\beta T = 1500$

$$M = [\underbrace{212}_{I=7} \quad \underbrace{164}_{I=9} \quad \underbrace{124}_{I=12} \quad \underbrace{74}_{I=20}]$$

For $\beta T = 2000$

$$M = [\underbrace{248}_{I=8} \quad \underbrace{220}_{I=9} \quad \underbrace{164}_{I=12} \quad \underbrace{98}_{I=20}]$$

Now we use these four values of I and four values of M for each value of βT to compute the four graphs of probability of bit error with five traces each. We will use the equations and methods developed in Appendix B to compute these graphs. Recall that

$$X = \underbrace{\mathcal{R}\, PT(\text{QUOT}_1)}_{\text{Term \#1}} + \underbrace{\mathcal{R}\, PT(\text{QUOT}_2)}_{\text{Term \#2}} + \cdots + \underbrace{\mathcal{R}\, PT(\text{QUOT}_{10})}_{\text{Term \#10}} \tag{304}$$

Realizing this, we program the ten terms given in Appendix B in the following way:

TERM #1 [Eq. (272)]: We will compute the QUOT_1 portion of this term four times, once for each value of βT = 500, 1000, 1500, and 2000. We will use each of these four values in a separate program to compute the probability of bit error according to the equations developed in Appendix B.

TERM #2 [Eq. (274)]: We will compute the QUOT_2 portion of TERM #2 four times, once for each value of βT = 500, 1000, 1500, and 2000. Each of these values will be used in a separate program to compute the probability of bit error according to the equations developed in Appendix B.

TERM #3 [Eq. (276)]: We will compute the QUOT_3 portion of TERM #3 four times, once each value of βT = 500, 1000, 1500, and 2000. Each of these values will be used in a separate program to compute the probability of bit error according to the equations developed in Appendix B.

TERM #4 [Eq. (277)]: We will compute the QUOT_4 portion of TERM #4 four times, once for each value of $\beta T = 500$, 1000, 1500, and 2000. Each of these values will be used in a separate program to compute the probability of bit error according to the equations developed in Appendix B.

TERM #5 [Eq. (282)]: We will compute the value of QUOT_5 a total of 48 times, 12 times for each value of $\beta T = 500$, 1000, 1500, and 2000. For each value of βT, we have the four corresponding values of the normalized channel spacing integer I, or equivalently, the number of adjacent channels M. We will compute the value of QUOT_5 three times for each value of I. QUOT_5 will be computed once for all $M/2$ of the lower adjacent channels being packed with 1s and the upper adjacent channels being packed with 0s. This is the case: $(b^- = 1, b^+ = 0)$. QUOT_5 will be computed once for the lower adjacent channels being packed with 0s and the upper adjacent channels being packed with 1s. This is the case: $(b^- = 0, b^+ = 1)$. Finally, we will compute QUOT_5 once more for both the lower and upper adjacent channels being packed with 1's. This is the case: $(b^- = 1, b^+ = 1)$. These twelve values of QUOT_5 for each value of βT will be used in each of the four separate bit error programs to compute each of the four multiple channel probability of bit error traces using the equations and methods developed in Appendix B.

TERM #6 [Eq. (283)]: We will compute the QUOT_6 portion of TERM #6 four times, once each for $\beta T = 500$, 1000, 1500, and 2000. Each of the values will be used in separate to compute program probability of bit error according to the equations developed in Appendix B.

TERM #7 [Eq. (284)]: We will compute the QUOT_7 portion of TERM #7 four times, once for each value of βT = 500, 1000, 1500, and 2000. Each of these values will be used in a separate program to compute the probability of bit error according to the equations developed in Appendix B.

TERM #8 [Eq. (292)]: We compute the value of QUOT_8 twelve times for each value of βT. These twelve values of QUOT_8 for each value of βT will be used in each of the four separate bit error graphing programs to compute each of the four multiple channel probability of bit error traces using the equations and methods developed in Appendix B.

TERM #9 [Eq. (296)]: We compute the value of QUOT_9 twelve times for each value of βT. These twelve values of QUOT_9 for each value of βT will be used in each of the four separate bit error graphing programs to compute each of the four multiple channel probability of bit error traces using the equations and methods developed in Appendix B.

TERM #10 [Eq. (299)]: We compute the value of QUOT_{10} twelve times for each value of βT. These twelve values of QUOT_{10} for each value of βT will be used in each of the four separate bit error graphing programs to compute each of the four multiple channel probability of bit error traces using the equations and methods developed in Appendix B.

Note: When we say we are computing QUOT_j, we are not being exactly mathematically correct, as we compute each of these terms without the bit values $b_{0,0}$ and/or $b_{0,-1}$ the factor $(1-\rho)^2$. These are accounted for in the final programs. For example

$$\text{Term } \#1 = \mathcal{R}\, PT \underbrace{(1-\rho)^2 b_{0,0}^2 \sum_{g=0}^{\beta T-1} \sum_{m=0}^{\beta T-1} \rho^{g+m} \left(1 - \left(\frac{\max(g,m)}{\beta T}\right)\right)}_{\text{QUOT}_1} \tag{305}$$

However, the computer program for Term #1 (QUOT_1) only computes

$$\sum_{g=0}^{\beta T-1} \sum_{m=0}^{\beta T-1} \rho^{g+m} \left(1 - \left(\frac{\max(g,m)}{\beta T}\right)\right) \tag{306}$$

four times for βT = 500, 1000, 1500, 2000. $(1-\rho)^2$ and the value of $(b_{0,0})^2$ are accounted for in the final programs which utilize the equations and methods developed in Appendix B. The author apologizes for the slight stretch of the truth, but it seemed necessary to succinctly explain the general method of computing each of the ten terms, and the final graphs.

For completness we also present the probability of bit error equation for a single channel operation

$$P_b = Q\left(\frac{1}{2}Z\right) \tag{307}$$

where we recall [Eq. (271), Appendix B] that

$$Z = \frac{\mathcal{R}\, PT}{\sqrt{N_0 T}} = \mathcal{R}\, P\sqrt{\frac{T}{N_0}} \tag{308}$$

and

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy \tag{309}$$

The computer programs for each of the ten terms, the numerical results, and the final graph are now presented.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                       THESIS COMPUTER WORK
%
%                            ---------
%                           |         |
%                           | TERM #1 |
%                           |_________|
%
%   COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE THE FOUR VALUES OF
%             TERM #1 AND TO WRITE THESE VALUES TO DIARY FILE FOR LATER
%             USE/MANIPULATION IN CALCULATING THE DETECTION STATISTIC
%             AND THE PROBABILITIES OF BIT ERROR
%
%
%   JOHN A. STUDER                    DATE LAST MODIFIED: 11 SEP 94
%   CPT, U.S ARMY
%   550-53-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

rho = 0.99;
betatau = [500 1000 1500 2000];
for i = 1:4
   term1sum(i)= 0;
    for g = 1:betatau(i)
       for m = 1:betatau(i)
         term1sum(i) = term1sum(i)+(rho.^((g-1)+(m-1))*...
                           (1-(max([g-1 m-1]')/betatau(i))));
        end
    end
end
diary johnman1.txt
diary on
term1sum
diary off
end
```

```
term1sum =

   1.0e+03 *

    7.0510    8.5126    9.0083    9.2563        % numbers/arrows/words
                                                % below added after matlab
                                                % dumped answers to file using
                                                % the diary command

      ^         ^         ^         ^

     500       1000      1500      2000   ----  betatau values

% The numbers displayed above are the computed values of
% Term #1 for the four values of betatau given above.
% These values will be used later in other programs to compute detection
% statistics and the probabilities of bit error for various signal to noise
% ratios.
```

```
 ------------
|            |
|  TERM #1   |
|____________|
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                         THESIS COMPUTER WORK
%
%
%                              ----------
%                             |          |
%                             |  TERM #2 |
%                             |__________|
%
%  COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE THE FOUR VALUES
%            OF TERM #2 CORRESPONDING TO THE FOUR VALUES OF
%            betatau: 500,1000,1500,2000. WE WILL USE BACKGROUND
%            PROCESSING TO WRITE THESE FOUR VALUES TO AN OUTPUT
%            FILE CALLED term2.out. WE WILL USE THESE FOUR OUTPUT
%            VALUES FOR TERM #2 TO LATER, IN ANOTHER PROGRAM
%            COMPUTE THE DETECTION STATISTICS AND PROBABILITIES
%            OF BIT ERROR FOR THE VARIOUS SIGNAL TO NOISE RATIOS.
%
%
%  JOHN A. STUDER                 DATE LAST MODIFIED: 11 SEP 94
%  CPT, U.S. ARMY
%  550-53-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
rho = 0.99;
betatau = [500 1000 1500 2000];
for i = 1:4
   term2sum(i) = 0;
   for g = 1:betatau(i)-1
      for m = 1: betatau(i)-1
          term2sum(i) = term2sum(i) + ((rho^(g+m))*min(g,m));
      end
   end
   term2sumfinal(i) = term2sum(i)/betatau(i);
end
term2sumfinal
exit
```

```
                          < M A T L A B (R) >
               (c) Copyright 1984-94 The MathWorks, Inc.
                          All Rights Reserved
                              Version 4.2
                              Mar 29 1994

Commands to get started: intro, demo, help help
Commands for more information: help, whatsnew, info, subscribe


term2sumfinal =                                   % The numbers/arrows/words
                                                  % below were added after
  959.5662  492.4271  328.3413  246.2563          % the matlab background job
                                                  % dumped the values to the
%    ^         ^         ^         ^              % file term2.out
%   500       1000      1500      2000            % All values/text not done by
%                ^                                % Sun Stn is preceded by a "%"
%    ------ betatau values--------


                         %%%%%%%%%%%%%%%
                         %             %
                         %   TERM #2   %
                         %             %
                         %%%%%%%%%%%%%%%


 29965020 flops.
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                       THESIS COMPUTER WORK
%
%
%
%                         ----------------
%                        |                |
%                        |    TERM #3     |
%                        |________________|
%
%  COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE THE FOUR VALUES OF
%            TERM #3 CORRESPONDING TO THE FOUR VALUES OF
%            betatau: 500,1000,1500,2000. WE WILL USE BACKGROUND
%            PROCESSING TO COMPUTE THE VALUES AND WRITE THESE FOUR
%            VALUES TO A FILE CALLED term3.out. WE WILL USE THESE
%            FOUR VALUES FOR TERM #3 TO LATER, IN ANOTHER PROGRAM
%            COMPUTE THE DETECTION STATISTICS AND PROBABILITIES OF
%            BIT ERROR FOR THE VARIOUS SIGNAL TO NOISE RATIOS.
%
%  JOHN A. STUDER                 DATE LAST MODIFIED: 12 SEP 1994
%  CPT, U.S. ARMY
%  550-53-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
format short e
rho = 0.99;
betatau = [500 1000 1500 2000];
for i = 1:4
   term3sum(i) = 0;
   for g = 1:betatau(i)-1
      for m = betatau(i):((2*betatau(i))-1)
         if m-betatau(i) < g
             term3sum(i) = term3sum(i) + ((rho^(g+m))*(g-(m-betatau(i))));
         end
      end
   end
term3sumfinal(i) = (2*term3sum(i))/betatau(i);
end
term3sumfinal
exit
```

```
                        < M A T L A B (R) >
                (c) Copyright 1984-93 The MathWorks, Inc.
                        All Rights Reserved
                            Version 4.1
                            Jun 15 1993

Commands to get started: intro, demo, help help
Commands for more information: help, whatsnew, info, subscribe


term3sumfinal =                                              % Numbers/arrows/words
                                                             % below were added after
   1.2211e+01   4.2917e-02   1.8815e-04   9.2720e-07         % the matlab background
                                                             % job dumped the values
   %    ^            ^             ^            ^            % to the file term3.out
   %   500         1000          1500         2000           % All values/text not
   %                                                         % done by Sun Stn. is
   %                                                         % preceded by a "%".
   %   ------------- betatau values ----------------


                     %%%%%%%%%%%%%%%
                     %             %
                     %   TERM #3   %
                     %             %
                     %%%%%%%%%%%%%%%



 29990004 flops.
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                            THESIS COMPUTER WORK
%
%
%                                 --------------
%                                |              |
%                                |   TERM #4    |
%                                |______________|
%
%  COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE THE FOUR VALUES
%            OF TERM #4 CORRESPONDING TO THE FOUR VALUES OF
%            betatau: 500,1000,1500,2000. WE WILL USE BACKGROUND
%            PROCESSING TO WRITE THESE FOUR VALUES TO AN OUTPUT
%            FILE CALLED term4.out. WE WILL USE THESE FOUR OUTPUT
%            VALUES FOR TERM #4 TO LATER , IN ANOTHER PROGRAM
%            COMPUTE THE DETECTION STATISTICS AND PROBABILITIES
%            OF BIT ERROR FOR THE VARIOUS SIGNAL TO NOISE RATIOS.
%
%
%  JOHN A. STUDER                      DATE LAST MODIFIED: 13 SEP 94
%  CPT, U.S. ARMY
%  550-53-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
format short e
rho = 0.99;
betatau = [500 1000 1500 2000];
for i = 1:4
   term4sum(i) = 0;
   for g = betatau(i):((2*betatau(i))-1)
      for m = betatau(i):((2*betatau(i))-1)
          term4sum(i) = term4sum(i) + ((rho^(g+m))*...
                         (1-(max(g-betatau(i),m-betatau(i))/betatau(i))));
      end
   end
   term4sumfinal(i) = term4sum(i);
end
term4sumfinal
exit
```

```
                              < M A T L A B (R) >
                  (c) Copyright 1984-94 The MathWorks, Inc.
                              All Rights Reserved
                                  Version 4.2
                                  Mar 29 1994

Commands to get started: intro, demo, help help
Commands for more information: help, whatsnew, info, subscribe


term4sumfinal =                                                % Numbers/arrows/words
                                                               % below were added after
   3.0440e-01   1.5865e-05   7.2482e-10   3.2152e-14           % the matlab bkgd job
                                                               % dumped the values to
  %     ^             ^            ^            ^              % the file term4.out
  %    500          1000         1500         2000             % All values/text not
  %                        ^                                   % done by Sun Stn is
  %                                                            % preceded by a "%".
  %     ------------- betatau values --------------


                         %%%%%%%%%%%%%%%%
                         %              %
                         %    TERM #4   %
                         %              %
                         %%%%%%%%%%%%%%%%


 60010008 flops.
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                  THESIS COMPUTER WORK
%
%
%
%
%                                  -------------------
%                                 |                   |
%                                 |      TERM #5      |
%                                 |   betatau = 500   |
%                                 |___________________|
%
%  COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE TERM #5
%            FOR THE VALUE OF betatau = 500. WE WILL COMPUTE
%            FOR THE FOUR VALUES OF I (AN INTEGER). THE VALUES OF I FOR
%            betatau = 500 are I = [4 5 8 20]. WE WILL COMPUTE THE VALUE OF
%            TERM #5 THREE TIMES FOR EACH CORRESPONDING VALUE OF I. ONCE FOR
%            ALL M/2 OF THE LOWER ACI CHANNELS (have negative summation index)
%            BEING PACKED WITH 1'S AND THE UPPER ACI CHANNELS ALL BEING PACKED
%            WITH 0'S. THIS IS THE CASE: (b-=1,b+=0). ONCE FOR ALL THE LOWER
%            ACI CHANNELS BEING PACKED WITH ZERO'S AND THE UPPER ACI CHANNELS
%            (have a positive summation index) BEING PACKED WITH 1'S. THIS
%            IS THE CASE: (b-=0,b+=1). FINALLY ONCE FOR BOTH THE UPPER AND
%            LOWER ACI CHANNELS BEING PACKED WITH 1'S. THIS IS THE CASE:
%            (b-=1,b+=1). WE WILL USE BACKGROUND PROCESSING WHICH WILL
%            DUMP ALL TWELVE VALUES TO A FILE CALLED term5500.mat
%            SINCE THE FILE IS IN BINARY FORMAT WE WILL USE MATLAB
%            INTERACTIVE COMMANDS AND THE TEXT EDITOR TO PRODUCE
%            READABLE HARDCOPY RESULTS. AFTER THIS IS ALL DONE, WE WILL
%            USE THESE VALUES LATER , IN ANOTHER PROGRAM TO COMPUTE THE
%            DETECTION STATISTICS AND PROBABILITIES OF BIT ERROR FOR THE
%            VARIOUS SIGNAL TO NOISE RATIOS.
%
%  JOHN A. STUDER                        DATE LAST MODIFIED: 22 OCT 94
%  CPT, U.S. ARMY
%  550-53-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
format short e
rho = 0.99;
betatau = 500;
I = [4 5 8 20];
M = [124 98 60 24];
g = 0:(betatau-1);
m = 0:(betatau-1);
row = rho.^g;
col = rho.^m;
rhomatrix = col'*row;
rowprime = g(ones(betatau,1),:);
colprime = rowprime';
WO = max(rowprime,colprime);
C = WO/betatau;
ONEMINUSMAX = 1-C;
```

```
A = colprime-rowprime;      % m-g for later complex exponential
for i = 1:4
   bitmatrix = [-M(i)/2:-1 zeros(1,M(i)/2);zeros(1,M(i)/2) 1:M(i)/2;...
                                                    -M(i)/2:-1 1:M(i)/2];
   const = ((2*pi*I(i))/betatau);
      for bitpat = 1:3
         term5sum(i,bitpat) = 0;
         work = bitmatrix(bitpat,:);
         m = work==0;
         work(m)=[];
         k = work;
         n = k;
         for kct = 1:length(k)
            for nct = 1:length(n)
               if kct == nct
                  B = j*((2*pi*k(kct)*I(i)*A)/betatau);
                  D = exp(B);
                  term5sum(i,bitpat) = term5sum(i,bitpat)+...
                               sum(sum(rhomatrix.*D.*ONEMINUSMAX));
               else
                  E1 = exp(j*const*((n(nct)*colprime)-(k(kct)*rowprime)));%%%%
                  E2 = 1-exp(j*const*(k(kct)-n(nct))*W0);
                  term5sum(i,bitpat) = term5sum(i,bitpat) +...
                   sum(sum((rhomatrix.*E1.*E2)/(j*2*pi*I(i)*(k(kct)-n(nct)))));
               end
            end
         end
        end
end
save term5500 term5sum
exit
```

```
                    < M A T L A B (R) >
          (c) Copyright 1984-93 The MathWorks, Inc.
                    All Rights Reserved
                        Version 4.1
                        Jun 15 1993

Commands to get started: intro, demo, help help
Commands for more information: help, whatsnew, info, subscribe

>> load term5500.mat
>> who

Your variables are:

term5sum

>> term5sum

term5sum =

   1.0e+03 *

   1.5423 - 0.0000i   1.5423 + 0.0000i   1.3825 + 0.0000i  % I = 4
   0.9419 - 0.0000i   0.9419 + 0.0000i   0.9001 - 0.0000i  % I = 5
   0.3307 - 0.0000i   0.3307 + 0.0000i   0.3576 + 0.0000i  % I = 8
   0.0431 - 0.0000i   0.0431 + 0.0000i   0.0580 - 0.0000i  % I = 20

>>
%           ^                  ^                  ^
% CASE: (b-=1,b+=0)  CASE: (b-=0,b+=1)   CASE: (b-=1,b+=1)


                  %%%%%%%%%%%%%%%%%%%
                  %     TERM #5     %
                  %  betatau = 500  %
                  %%%%%%%%%%%%%%%%%%%


     NOTE: THIS IS THE FILE term5500.out CREATED FROM THE
           BINARY FILE term5500.mat USING MATLAB INTERACTIVE COMMANDS.
           ALL TEXT PRECEDED BY A % WAS ADDED LATER USING A
           TEXT EDITOR.
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                        THESIS COMPUTER WORK
%
%
%
%                          ---------------
%                         |               |
%                         |    TERM #6    |
%                         |_______________|
%
%  COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE THE FOUR VALUES OF
%            TERM #6 CORRESPONDING TO THE FOUR VALUES OF betatau WHICH ARE:
%            500,1000,1500,2000. WE WILL USE BACKGROUND PROCESSING TO
%            COMPUTE THE VALUES AND WRITE THESE FOUR VALUES TO A FILE
%            CALLED term6.out. WE WILL USE THESE FOUR VALUES FOR TERM #6
%            TO LATER, IN ANOTHER PROGRAM COMPUTE THE DETECTION STATISTICS
%            AND PROBABILITIES OF BIT ERROR FOR THE VARIOUS SIGNAL TO
%            NOISE RATIOS.
%
%  JOHN A. STUDER                      DATE LAST MODIFIED: 16 SEP 94
%  CPT, U.S. ARMY
%  550-53-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
format short e
rho = 0.99;
betatau = [500 1000 1500 2000];
for i = 1:4
   term6sum(i) = 0;
   for g = 1:betatau(i)
      for m = 1:betatau(i)-1
         if (g-1) < m
            term6sum(i) = term6sum(i) + ((rho^((g-1)+m))*(m-(g-1)));
         end
      end
   end
term6sumfinal(i) = (2*term6sum(i))/betatau(i);
end
term6sumfinal
exit
```

```
                    < M A T L A B (R) >
          (c) Copyright 1984-94 The MathWorks, Inc.
                    All Rights Reserved
                        Version 4.2
                        Mar 29 1994

Commands to get started: intro, demo, help help
Commands for more information: help, whatsnew, info, subscribe


term6sumfinal =                                                  % Numbers/arrows/words
                                                                 % below were added after
   1.8585e+03   9.9411e+02   6.6331e+02   4.9749e+02            % the matlab bkgd job
                                                                 % dumped the values to
   %    ^            ^            ^            ^                 % the file term6.out
   %   500         1000         1500         2000                % All values/text not
   %                                                             % done by Sun Stn is
   %   ------------- betatau values --------------               % preceded by a "%".


                        %%%%%%%%%%%%%%%
                        %             %
                        %   TERM #6   %
                        %             %
                        %%%%%%%%%%%%%%%



 33732508 flops.
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                            THESIS COMPUTER WORK
%
%
%
%                               ----------------
%                              |                |
%                              |    TERM #7     |
%                              |________________|
%
%  COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE THE FOUR VALUES OF
%            TERM #7 CORRESPONDING TO THE FOUR VALUES OF betatau WHICH ARE
%            500,1000,1500,2000. WE WILL USE BACKGROUND PROCESSING TO COMPUTE
%            THE VALUES AND WRITE THESE FOUR VALUES TO A FILE CALLED
%            term7.out. WE WILL USE THESE FOUR VALUES FOR TERM #7 TO
%            LATER, IN ANOTHER PROGRAM COMPUTE THE DETECTION STATISTICS
%            AND PROBABILITIES OF BIT ERROR FOR THE VARIOUS SIGNAL TO
%            NOISE RATIOS.
%
%  JOHN A. STUDER                   DATE LAST MODIFIED: 16 SEP 94
%  CPT, U.S. ARMY
%  550-532-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
format short e
rho = 0.99;
betatau = [500 1000 1500 2000];
for i = 1:4
   term7sum(i) = 0;
   for g = 1:betatau(i)
     for m = betatau(i):((2*betatau(i))-1)
       term7sum(i) = term7sum(i) + ...
                     ((rho^((g-1)+m))*(1-(max(g-1,m-betatau(i))/betatau(i))));
     end
   end
term7sumfinal(i) = 2*term7sum(i);
end
term7sumfinal
exit
```

```
                              < M A T L A B (R) >
                    (c) Copyright 1984-94 The MathWorks, Inc.
                              All Rights Reserved
                                  Version 4.2
                                  Mar 29 1994

Commands to get started: intro, demo, help help
Commands for more information: help, whatsnew, info, subscribe

term7sumfinal =                                           % Numbers/arrows/words
                                                          % below were added after
   9.2657e+01   7.3500e-01   5.1105e-03   3.4503e-05      % the matlab bkgd job
                                                          % dumped the values to
   %    ^            ^            ^             ^         % the file term7.out
   %   500         1000         1500          2000        % All values/text not
   %                                                      % done by Sun Stn is
   %                                                      % preceded by a "%".
   %   ----------- betatau values ----------------


                        %%%%%%%%%%%%%%%
                        %             %
                        %   TERM #7   %
                        %             %
                        %%%%%%%%%%%%%%%



 67510004 flops.
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                          THESIS COMPUTER WORK
%
%
%
%
%                           ------------------
%                          |                  |
%                          |     TERM #8      |
%                          |  betatau = 500   |
%                          |__________________|
%
%  COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE TERM #8
%            FOR THE VALUE OF betatau = 500. WE WILL COMPUTE
%            FOR THE FOUR VALUES OF I (AN INTEGER). THE VALUES OF I FOR
%            betatau = 500 are I = [4 5 8 20]. WE WILL COMPUTE THE VALUE OF
%            TERM #8 THREE TIMES FOR EACH CORRESPONDING VALUE OF I. ONCE FOR
%            ALL M/2 OF THE LOWER ACI CHANNELS (have negative summation index)
%            BEING PACKED WITH 1'S AND THE UPPER ACI CHANNELS ALL BEING PACKED
%            WITH 0'S. THIS IS THE CASE: (b-=1,b+=0). ONCE FOR ALL THE LOWER
%            ACI CHANNELS BEING PACKED WITH ZERO'S AND THE UPPER ACI CHANNELS
%            (have a positive summation index) BEING PACKED WITH 1'S. THIS
%            IS THE CASE: (b-=0,b+=1). FINALLY ONCE FOR BOTH THE UPPER AND
%            LOWER ACI CHANNELS BEING PACKED WITH 1'S. THIS IS THE CASE:
%            (b-=1,b+=1). WE WILL USE BACKGROUND PROCESSING WHICH WILL
%            DUMP ALL TWELVE VALUES TO A FILE CALLED term8500.out. WE WILL
%            USE THESE VALUES LATER , IN ANOTHER PROGRAM TO COMPUTE THE
%            DETECTION STATISTICS AND PROBABILITIES OF BIT ERROR FOR THE
%            VARIOUS SIGNAL TO NOISE RATIOS.
%
%  JOHN A. STUDER                      DATE LAST MODIFIED: 26 SEP 94
%  CPT, U.S. ARMY
%  550-53-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
format short e
rho = 0.99;
betatau = 500;
I = [4 5 8 20];
M = [124 98 60 24];
for i = 1:4
   k = [-M(i)/2:-1 1:M(i)/2];
   term8sum500(i) = 0;
   for ct = 1:M(i)
      for g = 1:betatau
         for m = 1:betatau
            term8sum500(i) = term8sum500(i) + ((rho^((g-1)+(m-1)))/...
                             (pi*k(ct)*I(i)))*(sin(2*pi*k(ct)*I(i)*...
                             (1-((m-1)/betatau)))-sin(((2*pi*k(ct)*I(i))/...
                             betatau)*(max(g-1,m-1)-(m-1))));
         end
      end
      if ct == M(i)/2
```

```
            term8500left(i) = term8sum500(i);
         end
      end
      term8500lr(i) = term8sum500(i);
      term8500right(i) = term8500lr(i)-term8500left(i);
   end
   term8500left
   term8500right
   term8500lr
   exit
```

```
                          < M A T L A B (R) >
                (c) Copyright 1984-94 The MathWorks, Inc.
                          All Rights Reserved
                              Version 4.2
                              Mar 29 1994

Commands to get started: intro, demo, help help
Commands for more information: help, whatsnew, info, subscribe


term8500left =

  -3.7098e+02  -2.3852e+02  -9.2820e+01  -1.4109e+01  % CASE: (b-=1,b+=0)


term8500right =

  -3.7098e+02  -2.3852e+02  -9.2820e+01  -1.4109e+01  % CASE: (b-=0,b+=1)


term8500lr =

  -7.4196e+02  -4.7703e+02  -1.8564e+02  -2.8217e+01  % CASE: (b-=1,b+=1)

%      ^             ^             ^             ^
%    I = 4         I = 5         I = 8         I = 20
%    ---------------------------------------------
%                          ^
%   VALUES OF NORMALIZED CHANNEL SPACING INTEGER I


                       %%%%%%%%%%%%%%%%%%%%
                       %      TERM #8     %
                       %  betatau = 500   %
                       %%%%%%%%%%%%%%%%%%%%

                         NOTE: ALL TEXT PRECEDED BY A % WAS ADDED
                               BY A TEXT EDITOR AFTER THE BACKGROUND
                               JOB CREATED THE FILE term8500.out.


 2142000318 flops.
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                  THESIS COMPUTER WORK
%
%
%
%
%                                 ------------------
%                                |                  |
%                                |     TERM #8      |
%                                |  betatau = 1000  |
%                                |__________________|
%
%   COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE TERM #8
%             FOR THE VALUE OF betatau = 1000. WE WILL COMPUTE
%             FOR THE FOUR VALUES OF I (AN INTEGER). THE VALUES OF I FOR
%             betatau = 1000 are I = [5 6 9 20]. WE WILL COMPUTE THE VALUE OF
%             TERM #8 THREE TIMES FOR EACH CORRESPONDING VALUE OF I. ONCE FOR
%             ALL M/2 OF THE LOWER ACI CHANNELS (have negative summation index)
%             BEING PACKED WITH 1'S AND THE UPPER ACI CHANNELS ALL BEING PACKED
%             WITH 0'S. THIS IS THE CASE: (b-=1,b+=0). ONCE FOR ALL THE LOWER
%             ACI CHANNELS BEING PACKED WITH ZERO'S AND THE UPPER ACI CHANNELS
%             (have a positive summation index) BEING PACKED WITH 1'S. THIS
%             IS THE CASE: (b-=0,b+=1). FINALLY ONCE FOR BOTH THE UPPER AND
%             LOWER ACI CHANNELS BEING PACKED WITH 1'S. THIS IS THE CASE:
%             (b-=1,b+=1). WE WILL USE BACKGROUND PROCESSING WHICH WILL
%             DUMP ALL TWELVE VALUES TO A FILE CALLED term81k.mat
%             SINCE THE FILE IS IN A BINARY FORMAT WE WILL THEN USE
%             MATLAB INTERACTIVE COMMANDS AND THE TEXT EDITOR TO PRODUCE
%             READABLE HARDCOPY RESULTS. AFTER THIS IS DONE, WE WILL
%             USE THESE VALUES LATER , IN ANOTHER PROGRAM TO COMPUTE THE
%             DETECTION STATISTICS AND PROBABILITIES OF BIT ERROR FOR THE
%             VARIOUS SIGNAL TO NOISE RATIOS.
%
%   JOHN A. STUDER                       DATE LAST MODIFIED: 27 SEP 94
%   CPT, U.S. ARMY
%   550-53-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
format short e
rho = 0.99;
betatau = 1000;
I = [5 6 9 20];
M = [198 164 110 48];
for i = 1:4
   k = [-M(i)/2:-1 1:M(i)/2];
   term8sum(i) = 0;
   for ct = 1:M(i)
      for g = 1:betatau
         for m = 1:betatau
            term8sum(i) = term8sum(i) + ((rho^((g-1)+(m-1)))/...
                          (pi*k(ct)*I(i)))*(sin(2*pi*k(ct)*I(i)*...
                          (1-((m-1)/betatau))-sin(((2*pi*k(ct)*I(i))/...
                          betatau)*(max(g-1,m-1)-(m-1))));
```

```
      end
    end
    if ct == M(i)/2
      term81kleft(i) = term8sum(i);
    end
  end
  term81klr(i) = term8sum(i);
  term81kright(i) = term81klr(i)-term81kleft(i);
end
save term81k term81kleft term81kright term81klr
exit
```

```
                    < M A T L A B (R) >
        (c) Copyright 1984-93 The MathWorks, Inc.
                    All Rights Reserved
                        Version 4.1
                        Jun 15 1993

Commands to get started: intro, demo, help help
Commands for more information: help, whatsnew, info, subscribe

>> load term81k
>> who

Your variables are:

term81kleft    term81klr      term81kright

>> term81kleft

term81kleft =

 -464.3344 -327.7899 -148.2912  -29.7545    % CASE: (b-=1,b+=0)

>> term81kright

term81kright =

 -464.3344 -327.7899 -148.2912  -29.7545    % CASE: (b-=0,b+=1)

>> term81klr

term81klr =

 -928.6688 -655.5798 -296.5824  -59.5090    % CASE: (b-=1,b+=1)

>>
%       ^         ^         ^         ^
%   I = 5     I = 6     I = 9     I = 20
%   ------------------------------------
%                  ^
% VALUES OF THE NORMALIZED CHANNEL SPACING INTEGER I

                             %%%%%%%%%%%%%%%%%%
                             %     TERM #8    %
                             % betatau = 1000 %
                             %%%%%%%%%%%%%%%%%%

                       NOTE: THIS IS THE FILE term81k.out CREATED FROM THE
                             BINARY FILE term81k.mat USING MATLAB INTERACTIVE
                             COMMANDS. ALL TEXT PRECEDED BY A % WAS ADDED LATER
                             USING A TEXT EDITOR.
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                THESIS COMPUTER WORK
%
%
%
%
%                               -------------------
%                              |                   |
%                              |      TERM #8      |
%                              |  betatau = 1500   |
%                              |___________________|
%
% COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE TERM #8
%           FOR THE VALUE OF betatau = 1500. WE WILL COMPUTE
%           FOR THE FOUR VALUES OF I (AN INTEGER). THE VALUES OF I FOR
%           betatau = 1500 are I = [7 9 12 20]. WE WILL COMPUTE THE VALUE OF
%           TERM #8 THREE TIMES FOR EACH CORRESPONDING VALUE OF I. ONCE FOR
%           ALL M/2 OF THE LOWER ACI CHANNELS (have negative summation index)
%           BEING PACKED WITH 1'S AND THE UPPER ACI CHANNELS ALL BEING PACKED
%           WITH 0'S. THIS IS THE CASE: (b-=1,b+=0). ONCE FOR ALL THE LOWER
%           ACI CHANNELS BEING PACKED WITH ZERO'S AND THE UPPER ACI CHANNELS
%           (have a positive summation index) BEING PACKED WITH 1'S. THIS
%           IS THE CASE: (b-=0,b+=1). FINALLY ONCE FOR BOTH THE UPPER AND
%           LOWER ACI CHANNELS BEING PACKED WITH 1'S. THIS IS THE CASE:
%           (b-=1,b+=1). WE WILL USE BACKGROUND PROCESSING WHICH WILL
%           DUMP ALL TWELVE VALUES TO A FILE CALLED term815k.out WE WILL
%           USE THESE VALUES LATER, IN ANOTHER PROGRAM TO COMPUTE THE
%           DETECTION STATISTICS AND PROBABILITIES OF BIT ERROR FOR THE
%           VARIOUS SIGNAL TO NOISE RATIOS.
%
% JOHN A. STUDER                        DATE LAST MODIFIED: 27 SEP 94
% CPT, U.S. ARMY
% 550-53-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
format short e
rho = 0.99;
betatau = 1500;
I = [7 9 12 20];
M = [212 164 124 74];
for i = 1:4
  k = [-M(i)/2:-1 1:M(i)/2];
  term8sum(i) = 0;
  for ct = 1:M(i)
    for g = 1:betatau
      for m = 1:betatau
        term8sum(i) = term8sum(i) + ((rho^((g-1)+(m-1)))/...
                      (pi*k(ct)*I(i)))*(sin(2*pi*k(ct)*I(i)*...
                      (1-((m-1)/betatau)))-sin(((2*pi*k(ct)*I(i))/...
                      betatau)*(max(g-1,m-1)-(m-1))));
      end
    end
    if ct == M(i)/2
```

```
            term815kleft(i) = term8sum(i);
        end
    end
    term815klr(i) = term8sum(i);
    term815kright(i) = term815klr(i)-term815kleft(i);
end
term815kleft
term815kright
term815klr
exit
```

```
                        < M A T L A B (R) >
                (c) Copyright 1984-94 The MathWorks, Inc.
                        All Rights Reserved
                            Version 4.2
                            Mar 29 1994

Commands to get started: intro, demo, help help
Commands for more information: help, whatsnew, info, subscribe

malloc count matches malloc_debug_count
malloc count matches malloc_debug_count
malloc count matches malloc_debug_count
malloc count matches malloc_debug_count
malloc count matches malloc_debug_count
malloc count matches malloc_debug_count

term815kleft =

  -3.5253e+02  -2.1854e+02  -1.2475e+02  -4.5096e+01  % CASE: (b-=1,b+=0)


term815kright =

  -3.5253e+02  -2.1854e+02  -1.2475e+02  -4.5096e+01  % CASE: (b-=0,b+=1)


term815klr =

  -7.0505e+02  -4.3708e+02  -2.4950e+02  -9.0192e+01  % CASE: (b-=1,b+=1)

%      ^             ^             ^             ^
%     I = 7         I = 9         I = 12        I = 20
%     ----------------------------------------------
%                           ^
%    VALUES OF NORMALIZED CHANNEL SPACING INTEGER I

                        %%%%%%%%%%%%%%%%%%
                        %     TERM #8    %
                        % betatau = 1500 %
                        %%%%%%%%%%%%%%%%%%

                NOTE: THIS IS THE FILE term815k.out. ALL TEXT
                      PRECEDED BY A % WAS ADDED LATER USING A
                      TEXT EDITOR.

 36162000586 flops.
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                             THESIS COMPUTER WORK
%
%
%
%
%                            -------------------
%                           |                   |
%                           |      TERM #8      |
%                           |  betatau = 2000   |
%                           |___________________|
%
%  COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE TERM #8
%            FOR THE VALUE OF betatau = 2000. WE WILL COMPUTE
%            FOR THE FOUR VALUES OF I (AN INTEGER). THE VALUES OF I FOR
%            betatau = 2000 are I = [8 9 12 20]. WE WILL COMPUTE THE VALUE OF
%            TERM #8 THREE TIMES FOR EACH CORRESPONDING VALUE OF I. ONCE FOR
%            ALL M/2 OF THE LOWER ACI CHANNELS (have negative summation index)
%            BEING PACKED WITH 1'S AND THE UPPER ACI CHANNELS ALL BEING PACKED
%            WITH 0'S. THIS IS THE CASE: (b-=1,b+=0). ONCE FOR ALL THE LOWER
%            ACI CHANNELS BEING PACKED WITH ZERO'S AND THE UPPER ACI CHANNELS
%            (have a positive summation index) BEING PACKED WITH 1'S. THIS
%            IS THE CASE: (b-=0,b+=1). FINALLY ONCE FOR BOTH THE UPPER AND
%            LOWER ACI CHANNELS BEING PACKED WITH 1'S. THIS IS THE CASE:
%            (b-=1,b+=1). WE WILL USE BACKGROUND PROCESSING WHICH WILL
%            DUMP ALL TWELVE VALUES TO A FILE CALLED term82k.mat. WE WILL
%            THEN SINCE THE FILE IS IN BINARY FORMAT USE MATLAB INTERACTIVE
%            COMMANDS AND THE TEXT EDITOR TO PRODUCE READABLE HARDCOPY
%            RESULTS. WE WILL USE THESE VALUES LATER, IN ANOTHER PROGRAM
%            TO COMPUTE THE DETECTION STATISTICS AND PROBABILITIES OF
%            BIT ERROR FOR THE VARIOUS SIGNAL TO NOISE RATIOS.
%
%  JOHN A. STUDER                       DATE LAST MODIFIED: 27 SEP 94
%  CPT, U.S. ARMY
%  550-53-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
format short e
rho = 0.99;
betatau = 2000;
I = [8 9 12 20];
M = [248 220 164 98];
for i = 1:4
   k = [-M(i)/2:-1 1:M(i)/2];
   term8sum(i) = 0;
   for ct = 1:M(i)
      for g = 1:betatau
         for m = 1:betatau
            term8sum(i) = term8sum(i) + ((rho^((g-1)+(m-1)))/...
                          (pi*k(ct)*I(i)))*(sin(2*pi*k(ct)*I(i)*...
                          (1-((m-1)/betatau)))-sin(((2*pi*k(ct)*I(i))/...
                          betatau)*(max(g-1,m-1)-(m-1))));
         end
```

```
      end
      if ct == M(i)/2
         term82kleft(i) = term8sum(i);
      end
   end
   term82klr(i) = term8sum(i);
   term82kright(i) = term82klr(i)-term82kleft(i);
end
save term82k term82kleft term82kright term82klr
exit
```

```
                      < M A T L A B (R) >
            (c) Copyright 1984-93 The MathWorks, Inc.
                      All Rights Reserved
                          Version 4.1
                          Jun 15 1993

Commands to get started: intro, demo, help help
Commands for more information: help, whatsnew, info, subscribe

>> load term82k.mat
>> who

Your variables are:

term82kleft    term82klr     term82kright

>> term82kleft

term82kleft =

 -351.8931 -283.0055 -163.9044  -60.1542      % CASE: (b-=1,b+=0)

>> term82kright

term82kright =

 -351.8931 -283.0055 -163.9044  -60.1542      % CASE: (b-=0,b+=1)

>> term82klr

term82klr =

 -703.7863 -566.0110 -327.8088 -120.3084      % CASE: (b-=1,b+=1)
%     ^          ^        ^         ^
%  I = 8      I = 9    I = 12    I = 20
%
%  ------------------------------------
%                  ^
%  VALUES OF THE NORMALIZED CHANNEL SPACING INTEGER I


                         %%%%%%%%%%%%%%%%%%
                         %     TERM #8    %
                         % betatau = 2000 %
                         %%%%%%%%%%%%%%%%%%


                NOTE: THIS IS THE FILE term82k.out CREATED FROM THE
                      BINARY FILE term82k.mat USING MATLAB INTERACTIVE
                      COMMANDS. ALL TEXT PRECEDED BY A % WAS ADDED LATER
                      USING A TEXT EDITOR.
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               THESIS COMPUTER WORK
%
%
%
%
%                               -------------------
%                              |                   |
%                              |      TERM #9      |
%                              |   betatau = 500   |
%                              |___________________|
%
%  COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE TERM #9
%            FOR THE VALUE OF betatau = 500. WE WILL COMPUTE
%            FOR THE FOUR VALUES OF I (AN INTEGER). THE VALUES OF I FOR
%            betatau = 500 are I = [4 5 8 20]. WE WILL COMPUTE THE VALUE OF
%            TERM #9 THREE TIMES FOR EACH CORRESPONDING VALUE OF I. ONCE FOR
%            ALL M/2 OF THE LOWER ACI CHANNELS (have negative summation index)
%            BEING PACKED WITH 1'S AND THE UPPER ACI CHANNELS ALL BEING PACKED
%            WITH 0'S. THIS IS THE CASE: (b-=1,b+=0). ONCE FOR ALL THE LOWER
%            ACI CHANNELS BEING PACKED WITH ZERO'S AND THE UPPER ACI CHANNELS
%            (have a positive summation index) BEING PACKED WITH 1'S. THIS
%            IS THE CASE: (b-=0,b+=1). FINALLY ONCE FOR BOTH THE UPPER AND
%            LOWER ACI CHANNELS BEING PACKED WITH 1'S. THIS IS THE CASE:
%            (b-=1,b+=1). WE WILL USE BACKGROUND PROCESSING WHICH WILL
%            DUMP ALL TWELVE VALUES TO A FILE CALLED term9500.mat.
%            SINCE THE FILE IS IN A BINARY FORMAT WE WILL USE MATLAB
%            INTERACTIVE COMMANDS AND THE TEXT EDITOR TO PRODUCE READABLE
%            HARDCOPY RESULTS. AFTER THIS IS ALL DONE WE WILL
%            USE THESE VALUES LATER, IN ANOTHER PROGRAM TO COMPUTE THE
%            DETECTION STATISTICS AND PROBABILITIES OF BIT ERROR FOR THE
%            VARIOUS SIGNAL TO NOISE RATIOS.
%
%  JOHN A. STUDER                        DATE LAST MODIFIED: 28 SEP 94
%  CPT, U.S. ARMY
%  550-53-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
format short e
rho = 0.99;
betatau = 500;
I = [4 5 8 20];
M = [124 98 60 24];
for i = 1:4
   k = [-M(i)/2:-1 1:M(i)/2];
   term9sum(i) = 0;
   for ct = 1:M(i)
      for g = 1:betatau-1
         for m = 1:betatau
            if m-1 <  g
               term9sum(i) = term9sum(i) + ((rho^(g+(m-1)))/(pi*k(ct)...
                             *I(i)))*sin(2*pi*k(ct)*I(i)*((g-(m-1))...
                              /betatau));
```

```
            end
         end
      end
      if ct == M(i)/2
         term9500left(i) = term9sum(i);
      end
   end
   term9500lr(i) = term9sum(i);
   term9500right(i) = term9500lr(i)-term9500left(i);
end
save term9500 term9500left term9500right term9500lr
exit
```

```
<108 sp254201(SunOS) /kepler_u2/studer> matlab

                              < M A T L A B (R) >
                 (c) Copyright 1984-93 The MathWorks, Inc.
                              All Rights Reserved
                                  Version 4.1
                                  Jun 15 1993

Commands to get started: intro, demo, help help
Commands for more information: help, whatsnew, info, subscribe

>> load term9500.mat
>> who

Your variables are:

term9500left        term9500right
term9500lr

>> term9500left

term9500left =

  125.1608   80.4704   31.3156    4.7599    % CASE: (b-=1,b+=0)

>> term9500right

term9500right =

  125.1608   80.4704   31.3156    4.7599    % CASE: (b-=0,b+=1)

>> term9500lr

term9500lr =

  250.3216  160.9408   62.6313    9.5199    % CASE: (b-=1,b+=1)

>>
%     ^         ^         ^          ^
%   I = 4     I = 5     I = 8      I = 20
%   -------------------------------------
%                    ^
% VALUES OF NORMALIZED CHANNEL SPACING INTEGER I


                            %%%%%%%%%%%%%%%%%%%
                            %      TERM #9    %
                            %  betatau = 500  %
                            %%%%%%%%%%%%%%%%%%%
```

NOTE: THIS IS THE FILE term9500.out CREATED FROM THE BINARY FILE term9500.mat USING MATLAB INTERACTIVE COMMANDS. ALL TEXT PRECEDED BY A % WAS ADDED LATER USING A TEXT EDITOR.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                         THESIS COMPUTER WORK
%
%
%
%
%                        -------------------
%                        |                 |
%                        |     TERM #9     |
%                        |  betatau = 1000 |
%                        |_________________|
%
% COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE TERM #9
%           FOR THE VALUE OF betatau = 1000. WE WILL COMPUTE
%           FOR THE FOUR VALUES OF I (AN INTEGER). THE VALUES OF I FOR
%           betatau = 1000 are I = [5 6 9 20]. WE WILL COMPUTE THE VALUE OF
%           TERM #9 THREE TIMES FOR EACH CORRESPONDING VALUE OF I. ONCE FOR
%           ALL M/2 OF THE LOWER ACI CHANNELS (have negative summation index)
%           BEING PACKED WITH 1'S AND THE UPPER ACI CHANNELS ALL BEING PACKED
%           WITH 0'S. THIS IS THE CASE: (b-=1,b+=0). ONCE FOR ALL THE LOWER
%           ACI CHANNELS BEING PACKED WITH ZERO'S AND THE UPPER ACI CHANNELS
%           (have a positive summation index) BEING PACKED WITH 1'S. THIS
%           IS THE CASE: (b-=0,b+=1). FINALLY ONCE FOR BOTH THE UPPER AND
%           LOWER ACI CHANNELS BEING PACKED WITH 1'S. THIS IS THE CASE:
%           (b-=1,b+=1). WE WILL USE BACKGROUND PROCESSING WHICH WILL
%           DUMP ALL TWELVE VALUES TO A FILE CALLED term91k.mat.
%           SINCE THE FILE IS IN A BINARY FORMAT WE WILL USE MATLAB
%           INTERACTIVE COMMANDS AND THE TEXT EDITOR TO PRODUCE READABLE
%           HARDCOPY RESULTS. AFTER THIS IS ALL DONE WE WILL
%           USE THESE VALUES LATER, IN ANOTHER PROGRAM TO COMPUTE THE
%           DETECTION STATISTICS AND PROBABILITIES OF BIT ERROR FOR THE
%           VARIOUS SIGNAL TO NOISE RATIOS.
%
% JOHN A. STUDER                  DATE LAST MODIFIED: 28 SEP 94
% CPT, U.S. ARMY
% 550-53-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
format short e
rho = 0.99;
betatau = 1000;
I = [5 6 9 20];
M = [198 164 110 48];
for i = 1:4
   k = [-M(i)/2:-1 1:M(i)/2];
   term9sum(i) = 0;
   for ct = 1:M(i)
      for g = 1:betatau-1
         for m = 1:betatau
            if m-1 <  g
               term9sum(i) = term9sum(i) + ((rho^(g+(m-1)))/(pi*k(ct)...
                             *I(i)))*sin(2*pi*k(ct)*I(i)*((g-(m-1))...
                              /betatau));
```

```
            end
         end
      end
      if ct == M(i)/2
         term91kleft(i) = term9sum(i);
      end
   end
   term91klr(i) = term9sum(i);
   term91kright(i) = term91klr(i)-term91kleft(i);
end
save term91k term91kleft term91kright term91klr
exit
```

```
                      < M A T L A B (R) >
          (c) Copyright 1984-93 The MathWorks, Inc.
                      All Rights Reserved
                          Version 4.1
                          Jun 15 1993

Commands to get started: intro, demo, help help
Commands for more information: help, whatsnew, info, subscribe

>> load term91k.mat
>> who

Your variables are:

term91kleft    term91klr      term91kright

>> term91kleft

term91kleft =

  155.3047  109.6350   49.5986    9.9519       % CASE: (b-=1,b+=0)

>> term91kright

term91kright =

  155.3047  109.6350   49.5986    9.9519       % CASE: (b-=0,b+=1)

>> term91klr

term91klr =

  310.6094  219.2701   99.1971   19.9038       % CASE: (b-=1,b+=1)

>>
%     ^         ^         ^         ^
%   I = 5     I = 6     I = 9     I = 20
%   ------------------------------------
%                  ^
% VALUES OF THE NORMALIZED CHANNEL SPACING INTEGER I

                      %%%%%%%%%%%%%%%%%%
                      %     TERM #9    %
                      % betatau = 1000 %
                      %%%%%%%%%%%%%%%%%%

                NOTE: THIS IS THE FILE term91k.out CREATED FROM THE
                      BINARY FILE term91k.mat USING MATLAB INTERACTIVE
                      COMMANDS. ALL TEXT PRECEDED BY A % WAS ADDED LATER
                      USING A TEXT EDITOR.
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                              THESIS COMPUTER WORK
%
%
%
%
%                              -------------------
%                             |                   |
%                             |      TERM #9      |
%                             |  betatau = 1500   |
%                             |___________________|
%
%  COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE TERM #9
%            FOR THE VALUE OF betatau = 1500. WE WILL COMPUTE
%            FOR THE FOUR VALUES OF I (AN INTEGER). THE VALUES OF I FOR
%            betatau = 1500 are I = [7 9 12 20]. WE WILL COMPUTE THE VALUE OF
%            TERM #9 THREE TIMES FOR EACH CORRESPONDING VALUE OF I. ONCE FOR
%            ALL M/2 OF THE LOWER ACI CHANNELS (have negative summation index)
%            BEING PACKED WITH 1'S AND THE UPPER ACI CHANNELS ALL BEING PACKED
%            WITH 0'S. THIS IS THE CASE: (b-=1,b+=0). ONCE FOR ALL THE LOWER
%            ACI CHANNELS BEING PACKED WITH ZERO'S AND THE UPPER ACI CHANNELS
%            (have a positive summation index) BEING PACKED WITH 1'S. THIS
%            IS THE CASE: (b-=0,b+=1). FINALLY ONCE FOR BOTH THE UPPER AND
%            LOWER ACI CHANNELS BEING PACKED WITH 1'S. THIS IS THE CASE:
%            (b-=1,b+=1). WE WILL USE BACKGROUND PROCESSING WHICH WILL
%            DUMP ALL TWELVE VALUES TO A FILE CALLED term915k.mat.
%            SINCE THE FILE IS IN A BINARY FORMAT WE WILL USE MATLAB
%            INTERACTIVE COMMANDS AND THE TEXT EDITOR TO PRODUCE READABLE
%            HARDCOPY RESULTS. AFTER THIS IS ALL DONE WE WILL
%            USE THESE VALUES LATER, IN ANOTHER PROGRAM TO COMPUTE THE
%            DETECTION STATISTICS AND PROBABILITIES OF BIT ERROR FOR THE
%            VARIOUS SIGNAL TO NOISE RATIOS.
%
%  JOHN A. STUDER                      DATE LAST MODIFIED: 28 SEP 94
%  CPT, U.S. ARMY
%  550-53-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
format short e
rho = 0.99;
betatau = 1500;
I = [7 9 12 20];
M = [212 164 124 74];
for i = 1:4
   k = [-M(i)/2:-1 1:M(i)/2];
   term9sum(i) = 0;
   for ct = 1:M(i)
      for g = 1:betatau-1
         for m = 1:betatau
            if m-1 <  g
               term9sum(i) = term9sum(i) + ((rho^(g+(m-1)))/(pi*k(ct)...
                             *I(i)))*sin(2*pi*k(ct)*I(i)*((g-(m-1))...
                              /betatau));
```

```
            end
         end
      end
      if ct == M(i)/2
         term915kleft(i) = term9sum(i);
      end
   end
   term915klr(i) = term9sum(i);
   term915kright(i) = term915klr(i)-term915kleft(i);
end
save term915k term915kleft term915kright term915klr
exit
```

```
                        < M A T L A B (R) >
              (c) Copyright 1984-93 The MathWorks, Inc.
                        All Rights Reserved
                            Version 4.1
                            Jun 15 1993

Commands to get started: intro, demo, help help
Commands for more information: help, whatsnew, info, subscribe

>> load term915k
>> who

Your variables are:

term915kleft        term915kright
term915klr

>> term915kleft

term915kleft =

  117.9019   73.0900   41.7221   15.0822   % CASE: (b-=1,b+=0)

>> term915kright

term915kright =

  117.9019   73.0900   41.7221   15.0822   % CASE: (b-=0,b+=1)

>> term915klr

term915klr =

  235.8037  146.1800   83.4441   30.1644   % CASE: (b-=1,b+=1)

%      ^         ^         ^         ^
%    I = 7     I = 9     I = 12    I = 20
%    ------------------------------------
%                    ^
%   VALUES OF NORMALIZED CHANNEL SPACING INTEGER I

                          %%%%%%%%%%%%%%%%%%%
                          %     TERM #9     %
                          %  betatau = 1500 %
                          %%%%%%%%%%%%%%%%%%%

                NOTE: THIS IS THE FILE term915k.out CREATED FROM THE
                      BINARY FILE term915k.mat USING MATLAB INTERACTIVE
                      COMMANDS. ALL TEXT PRECEDED BY A % WAS ADDED LATER
                      USING A TEXT EDITOR.
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                             THESIS COMPUTER WORK
%
%
%
%
%                           -------------------
%                          |                   |
%                          |      TERM #9      |
%                          |  betatau = 2000   |
%                          |___________________|
%
%  COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE TERM #9
%            FOR THE VALUE OF betatau = 2000. WE WILL COMPUTE
%            FOR THE FOUR VALUES OF I (AN INTEGER). THE VALUES OF I FOR
%            betatau = 2000 are I = [8 9 12 20]. WE WILL COMPUTE THE VALUE OF
%            TERM #9 THREE TIMES FOR EACH CORRESPONDING VALUE OF I. ONCE FOR
%            ALL M/2 OF THE LOWER ACI CHANNELS (have negative summation index)
%            BEING PACKED WITH 1'S AND THE UPPER ACI CHANNELS ALL BEING PACKED
%            WITH 0'S. THIS IS THE CASE: (b-=1,b+=0). ONCE FOR ALL THE LOWER
%            ACI CHANNELS BEING PACKED WITH ZERO'S AND THE UPPER ACI CHANNELS
%            (have a positive summation index) BEING PACKED WITH 1'S. THIS
%            IS THE CASE: (b-=0,b+=1). FINALLY ONCE FOR BOTH THE UPPER AND
%            LOWER ACI CHANNELS BEING PACKED WITH 1'S. THIS IS THE CASE:
%            (b-=1,b+=1). WE WILL USE BACKGROUND PROCESSING WHICH WILL
%            DUMP ALL TWELVE VALUES TO A FILE CALLED term92k.mat.
%            SINCE THE FILE IS IN A BINARY FORMAT WE WILL USE MATLAB
%            INTERACTIVE COMMANDS AND THE TEXT EDITOR TO PRODUCE READABLE
%            HARDCOPY RESULTS. AFTER THIS IS ALL DONE WE WILL
%            USE THESE VALUES LATER, IN ANOTHER PROGRAM TO COMPUTE THE
%            DETECTION STATISTICS AND PROBABILITIES OF BIT ERROR FOR THE
%            VARIOUS SIGNAL TO NOISE RATIOS.
%
%  JOHN A. STUDER                        DATE LAST MODIFIED: 28 SEP 94
%  CPT, U.S. ARMY
%  550-53-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
format short e
rho = 0.99;
betatau = 1500;
I = [8 9 12 20];
M = [248 220 164 98];
for i = 1:4
   k = [-M(i)/2:-1 1:M(i)/2];
   term9sum(i) = 0;
   for ct = 1:M(i)
      for g = 1:betatau-1
         for m = 1:betatau
            if m-1 <  g
               term9sum(i) = term9sum(i) + ((rho^(g+(m-1)))/(pi*k(ct)...
                             *I(i)))*sin(2*pi*k(ct)*I(i)*((g-(m-1))...
                              /betatau));
```

```
            end
         end
      end
      if ct == M(i)/2
         term92kleft(i) = term9sum(i);
      end
   end
   term92klr(i) = term9sum(i);
   term92kright(i) = term92klr(i)-term92kleft(i);
end
save term92k term92kleft term92kright term92klr
exit
```

```
                     < M A T L A B (R) >
           (c) Copyright 1984-93 The MathWorks, Inc.
                     All Rights Reserved
                         Version 4.1
                         Jun 15 1993

Commands to get started: intro, demo, help help
Commands for more information: help, whatsnew, info, subscribe

>> load term92k.mat
>> who

Your variables are:

term92kleft    term92klr     term92kright

>> term92kleft

term92kleft =

   91.5258   73.0261   41.6758   15.0544        % CASE: (b-=1,b+=0)

>> term92kright

term92kright =

   91.5258   73.0261   41.6758   15.0544        % CASE: (b-=0,b+=1)

>> term92klr

term92klr =

  183.0515  146.0523   83.3516   30.1088        % CASE: (b-=1,b+=1)

>>
%   ^          ^        ^         ^
%  I = 8      I = 9    I = 12    I = 20
%
%  ------------------------------------
%                ^
%  VALUES OF THE NORMALIZED CHANNEL SPACING INTEGER I

                    %%%%%%%%%%%%%%%%%%
                    %     TERM #9    %
                    % betatau = 2000 %
                    %%%%%%%%%%%%%%%%%%

              NOTE: THIS IS THE FILE term92k.out CREATED FROM THE
                    BINARY FILE term92k.mat USING MATLAB INTERACTIVE
                    COMMANDS. ALL TEXT PRECEDED BY A % WAS ADDED LATER
                    USING A TEXT EDITOR.
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                 THESIS COMPUTER WORK
%
%
%
%
%                                  ------------------
%                                 |                  |
%                                 |     TERM #10     |
%                                 |  betatau = 500   |
%                                 |__________________|
%
%  COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE TERM #10
%            FOR THE VALUE OF betatau = 500. WE WILL COMPUTE
%            FOR THE FOUR VALUES OF I (AN INTEGER). THE VALUES OF I FOR
%            betatau = 500 are I = [4 5 8 20]. WE WILL COMPUTE THE VALUE OF
%            TERM #10 THREE TIMES FOR EACH CORRESPONDING VALUE OF I. ONCE FOR
%            ALL M/2 OF THE LOWER ACI CHANNELS (have negative summation index)
%            BEING PACKED WITH 1'S AND THE UPPER ACI CHANNELS ALL BEING PACKED
%            WITH 0'S. THIS IS THE CASE: (b-=1,b+=0). ONCE FOR ALL THE LOWER
%            ACI CHANNELS BEING PACKED WITH ZERO'S AND THE UPPER ACI CHANNELS
%            (have a positive summation index) BEING PACKED WITH 1'S. THIS
%            IS THE CASE: (b-=0,b+=1). FINALLY ONCE FOR BOTH THE UPPER AND
%            LOWER ACI CHANNELS BEING PACKED WITH 1'S. THIS IS THE CASE:
%            (b-=1,b+=1). WE WILL USE BACKGROUND PROCESSING WHICH WILL
%            DUMP ALL TWELVE VALUES TO A FILE CALLED term10500.mat.
%            SINCE THE FILE IS IN A BINARY FORMAT, WE WILL USE MATLAB
%            INTERACTIVE COMMANDS AND THE TEXT EDITOR TO PRODUCE READABLE
%            HARDCOPY RESULTS. AFTER THIS IS ALL DONE, WE WILL
%            USE THESE VALUES LATER , IN ANOTHER PROGRAM, TO COMPUTE THE
%            DETECTION STATISTICS AND PROBABILITIES OF BIT ERROR FOR THE
%            VARIOUS SIGNAL TO NOISE RATIOS.
%
%  JOHN A. STUDER                        DATE LAST MODIFIED: 29 SEP 94
%  CPT, U.S. ARMY
%  550-53-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
format short e
rho = 0.99;
betatau = 500;
I = [4 5 8 20];
M = [124 98 60 24];
for i = 1:4
   k = [-M(i)/2:-1 1:M(i)/2];
   term10sum(i) = 0;
   for ct = 1:M(i)
      for g = betatau:(2*betatau)-1
         for m = 1:betatau
            term10sum(i) = term10sum(i) + ((rho^(g+(m-1)))/...
                           (pi*k(ct)*I(i)))*(sin(2*pi*k(ct)*I(i)*...
                           (1-((m-1)/betatau)))-sin(((2*pi*k(ct)*I(i))/...
                           betatau)*(max(g-betatau,m-1)-(m-1))));
```

```
         end
      end
      if ct == M(i)/2
         term10500left(i) = term10sum(i);
      end
   end
   term10500lr(i) = term10sum(i);
   term10500right(i) = term10500lr(i)-term10500left(i);
end
save term10500 term10500left term10500right term10500lr
exit
```

```
                        < M A T L A B (R) >
              (c) Copyright 1984-93 The MathWorks, Inc.
                         All Rights Reserved
                             Version 4.1
                             Jun 15 1993

Commands to get started: intro, demo, help help
Commands for more information: help, whatsnew, info, subscribe

>> load term10500.mat
>> who

Your variables are:

term10500left       term10500right
term10500lr

>> term10500left

term10500left =

   -2.4375   -1.5672   -0.6099   -0.0927   % CASE: (b-=1,b+=0)

>> term10500right

term10500right =

   -2.4375   -1.5672   -0.6099   -0.0927   % CASE: (b-=0,b+=1)

>> term10500lr

term10500lr =

   -4.8750   -3.1343   -1.2197   -0.1854   % CASE: (b-=1,b+=1)

>>
%      ^         ^         ^         ^
%    I = 4     I = 5     I = 8     I = 20
%    ------------------------------------
%                     ^
% VALUES OF NORMALIZED CHANNEL SPACING INTEGER I

                           %%%%%%%%%%%%%%%%%%%
                           %      TERM #10   %
                           %  betatau = 500  %
                           %%%%%%%%%%%%%%%%%%%
       NOTE: THIS IS THE FILE term10500.out CREATED FROM THE BINARY FILE
             term10500.mat USING MATLAB INTERACTIVE COMMANDS.  ALL TEXT
             PRECEDED BY A % WAS ADDED LATER USING A TEXT EDITOR.
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                              THESIS COMPUTER WORK
%
%
%
%
%                              --------------------
%                              |                  |
%                              |     TERM #10     |
%                              | betatau = 1000   |
%                              |__________________|
%
%  COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE TERM #10
%            FOR THE VALUE OF betatau = 1000. WE WILL COMPUTE
%            FOR THE FOUR VALUES OF I (AN INTEGER). THE VALUES OF I FOR
%            betatau = 1000 are I = [5 6 9 20]. WE WILL COMPUTE THE VALUE OF
%            TERM #10 THREE TIMES FOR EACH CORRESPONDING VALUE OF I. ONCE FOR
%            ALL M/2 OF THE LOWER ACI CHANNELS (have negative summation index)
%            BEING PACKED WITH 1'S AND THE UPPER ACI CHANNELS ALL BEING PACKED
%            WITH 0'S. THIS IS THE CASE: (b-=1,b+=0). ONCE FOR ALL THE LOWER
%            ACI CHANNELS BEING PACKED WITH ZERO'S AND THE UPPER ACI CHANNELS
%            (have a positive summation index) BEING PACKED WITH 1'S. THIS
%            IS THE CASE: (b-=0,b+=1). FINALLY ONCE FOR BOTH THE UPPER AND
%            LOWER ACI CHANNELS BEING PACKED WITH 1'S. THIS IS THE CASE:
%            (b-=1,b+=1). WE WILL USE BACKGROUND PROCESSING WHICH WILL
%            DUMP ALL TWELVE VALUES TO A FILE CALLED term101k.mat.
%            SINCE THE FILE IS IN A BINARY FORMAT, WE WILL USE MATLAB
%            INTERACTIVE COMMANDS AND THE TEXT EDITOR TO PRODUCE READABLE
%            HARDCOPY RESULTS. AFTER THIS IS ALL DONE, WE WILL
%            USE THESE VALUES LATER , IN ANOTHER PROGRAM, TO COMPUTE THE
%            DETECTION STATISTICS AND PROBABILITIES OF BIT ERROR FOR THE
%            VARIOUS SIGNAL TO NOISE RATIOS.
%
%  JOHN A. STUDER                     DATE LAST MODIFIED: 29 SEP 94
%  CPT, U.S. ARMY
%  550-53-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
format short e
rho = 0.99;
betatau = 1000;
I = [5 6 9 20];
M = [198 164 110 48];
for i = 1:4
   k = [-M(i)/2:-1 1:M(i)/2];
   term10sum(i) = 0;
   for ct = 1:M(i)
      for g = betatau:(2*betatau)-1
         for m = 1:betatau
            term10sum(i) = term10sum(i) + ((rho^(g+(m-1)))/...
                           (pi*k(ct)*I(i)))*(sin(2*pi*k(ct)*I(i)*...
                           (1-((m-1)/betatau)))-sin(((2*pi*k(ct)*I(i))/...
                           betatau)*(max(g-betatau,m-1)-(m-1))));
```

```
         end
      end
      if ct == M(i)/2
         term101kleft(i) = term10sum(i);
      end
   end
   term101klr(i) = term10sum(i);
   term101kright(i) = term101klr(i)-term101kleft(i);
end
save term101k term101kleft term101kright term101klr
exit
```

```
                    (c) Copyright 1984-93 The MathWorks, Inc.
                              All Rights Reserved
                                  Version 4.1
                                  Jun 15 1993

Commands to get started: intro, demo, help help
Commands for more information: help, whatsnew, info, subscribe

>> load term101k.mat
>> who

Your variables are:

term101kleft        term101kright
term101klr

>> term101kleft

term101kleft =

   -0.0200   -0.0142   -0.0064   -0.0013    % CASE: (b-=1,b+=0)

>> term101kright

term101kright =

   -0.0200   -0.0142   -0.0064   -0.0013    % CASE: (b-=0,b+=1)

>> term101klr

term101klr =

   -0.0401   -0.0283   -0.0128   -0.0026    % CASE: (b-=1,b+=1)

>>
%    ^         ^         ^         ^
%  I = 5     I = 6     I = 9     I = 20
%
%  ------------------------------------
%                  ^
% VALUES OF THE NORMALIZED CHANNEL SPACING INTEGER I

                              %%%%%%%%%%%%%%%%%%
                              %    TERM #10    %
                              % betatau = 1000 %
                              %%%%%%%%%%%%%%%%%%

               NOTE: THIS IS THE FILE term101k.out CREATED FROM THE
                     BINARY FILE term101.mat USING MATLAB INTERACTIVE
                     COMMANDS. ALL TEXT PRECEDED BY A % WAS ADDED LATER
                     USING A TEXT EDITOR.
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                THESIS COMPUTER WORK
%
%
%                               ------------------
%                               |                |
%                               |    TERM #10    |
%                               | betatau = 1500 |
%                               |________________|
%
%  COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE TERM #10
%            FOR THE VALUE OF betatau = 1500. WE WILL COMPUTE
%            FOR THE FOUR VALUES OF I (AN INTEGER). THE VALUES OF I FOR
%            betatau = 1500 are I = [7 9 12 20]. WE WILL COMPUTE THE VALUE OF
%            TERM #10 THREE TIMES FOR EACH CORRESPONDING VALUE OF I. ONCE FOR
%            ALL M/2 OF THE LOWER ACI CHANNELS (have negative summation index)
%            BEING PACKED WITH 1'S AND THE UPPER ACI CHANNELS ALL BEING PACKED
%            WITH 0'S. THIS IS THE CASE: (b-=1,b+=0). ONCE FOR ALL THE LOWER
%            ACI CHANNELS BEING PACKED WITH ZERO'S AND THE UPPER ACI CHANNELS
%            (have a positive summation index) BEING PACKED WITH 1'S. THIS
%            IS THE CASE: (b-=0,b+=1). FINALLY ONCE FOR BOTH THE UPPER AND
%            LOWER ACI CHANNELS BEING PACKED WITH 1'S. THIS IS THE CASE:
%            (b-=1,b+=1). WE WILL USE BACKGROUND PROCESSING WHICH WILL
%            DUMP ALL TWELVE VALUES TO A FILE CALLED term1015k.mat.
%            SINCE THE FILE IS IN A BINARY FORMAT, WE WILL USE MATLAB
%            INTERACTIVE COMMANDS AND THE TEXT EDITOR TO PRODUCE READABLE
%            HARDCOPY RESULTS. AFTER THIS IS ALL DONE, WE WILL
%            USE THESE VALUES LATER , IN ANOTHER PROGRAM, TO COMPUTE THE
%            DETECTION STATISTICS AND PROBABILITIES OF BIT ERROR FOR THE
%            VARIOUS SIGNAL TO NOISE RATIOS.
%
%  JOHN A. STUDER                      DATE LAST MODIFIED: 29 SEP 94
%  CPT, U.S. ARMY
%  550-53-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
format short e
rho = 0.99;
betatau = 1500;
I = [7 9 12 20];
M = [212 164 124 74];
for i = 1:4
   k = [-M(i)/2:-1 1:M(i)/2];
   term10sum(i) = 0;
   for ct = 1:M(i)
      for g = betatau:(2*betatau)-1
         for m = 1:betatau
            term10sum(i) = term10sum(i) + ((rho^(g+(m-1)))/...
                           (pi*k(ct)*I(i)))*(sin(2*pi*k(ct)*I(i)*...
                           (1-((m-1)/betatau)))-sin(((2*pi*k(ct)*I(i))/...
                           betatau)*(max(g-betatau,m-1)-(m-1))));
         end
      end
```

```
      if ct == M(i)/2
         term1015kleft(i) = term10sum(i);
      end
   end
   term1015klr(i) = term10sum(i);
   term1015kright(i) = term1015klr(i)-term1015kleft(i);
end
save term1015k term1015kleft term1015kright term1015klr
exit
```

```
                              < M A T L A B (R) >
                  (c) Copyright 1984-93 The MathWorks, Inc.
                              All Rights Reserved
                                  Version 4.1
                                  Jun 15 1993

Commands to get started: intro, demo, help help
Commands for more information: help, whatsnew, info, subscribe

>> load term1015k.mat
>> who

Your variables are:

term1015kleft       term1015kright
term1015klr

>> term1015kleft

term1015kleft =

   1.0e-04 *

   -1.0000    -0.6199    -0.3539    -0.1279    % CASE: (b-=1,b+=0)

>> term1015kright

term1015kright =

   1.0e-04 *

   -1.0000    -0.6199    -0.3539    -0.1279    % CASE: (b-=0,b+=1)

>> term1015klr

term1015klr =

   1.0e-03 *

   -0.2000    -0.1240    -0.0708    -0.0256    % CASE: (b-=1,b+=1)

>>
%     ^         ^         ^         ^
%   I = 7     I = 9     I = 12    I = 20
%  ------------------------------------
%                  ^
%  VALUES OF NORMALIZED CHANNEL SPACING INTEGER I
```

```
%%%%%%%%%%%%%%%%%%
%      TERM #10      %
%  betatau = 1500 %
%%%%%%%%%%%%%%%%%%
```

NOTE: THIS IS THE FILE term1015k.out CREATED FROM THE BINARY FILE term1015k.mat USING MATLAB INTERACTIVE COMMANDS. ALL TEXT PRECEDED BY A % WAS ADDED LATER USING A TEXT EDITOR.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                THESIS COMPUTER WORK
%
%
%
%
%                                 -------------------
%                                |                   |
%                                |     TERM #10      |
%                                |  betatau = 2000   |
%                                |___________________|
%
%  COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE TERM #10
%            FOR THE VALUE OF betatau = 2000. WE WILL COMPUTE
%            FOR THE FOUR VALUES OF I (AN INTEGER). THE VALUES OF I FOR
%            betatau = 2000 are I = [8 9 12 20]. WE WILL COMPUTE THE VALUE OF
%            TERM #10 THREE TIMES FOR EACH CORRESPONDING VALUE OF I. ONCE FOR
%            ALL M/2 OF THE LOWER ACI CHANNELS (have negative summation index)
%            BEING PACKED WITH 1'S AND THE UPPER ACI CHANNELS ALL BEING PACKED
%            WITH 0'S. THIS IS THE CASE: (b-=1,b+=0). ONCE FOR ALL THE LOWER
%            ACI CHANNELS BEING PACKED WITH ZERO'S AND THE UPPER ACI CHANNELS
%            (have a positive summation index) BEING PACKED WITH 1'S. THIS
%            IS THE CASE: (b-=0,b+=1). FINALLY ONCE FOR BOTH THE UPPER AND
%            LOWER ACI CHANNELS BEING PACKED WITH 1'S. THIS IS THE CASE:
%            (b-=1,b+=1). WE WILL USE BACKGROUND PROCESSING WHICH WILL
%            DUMP ALL TWELVE VALUES TO A FILE CALLED term102k.mat.
%            SINCE THE FILE IS IN A BINARY FORMAT, WE WILL USE MATLAB
%            INTERACTIVE COMMANDS AND THE TEXT EDITOR TO PRODUCE READABLE
%            HARDCOPY RESULTS. AFTER THIS IS ALL DONE, WE WILL
%            USE THESE VALUES LATER , IN ANOTHER PROGRAM, TO COMPUTE THE
%            DETECTION STATISTICS AND PROBABILITIES OF BIT ERROR FOR THE
%            VARIOUS SIGNAL TO NOISE RATIOS.
%
%  JOHN A. STUDER                        DATE LAST MODIFIED: 28 NOV 94
%  CPT, U.S. ARMY
%  550-53-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
format short e
rho = 0.99;
betatau = 2000;
I = [8 9 12 20];
M = [248 220 164 98];
for i = 1:4
   k = [-M(i)/2:-1 1:M(i)/2];
   term10sum(i) = 0;
   for ct = 1:M(i)
      for g = betatau:(2*betatau)-1
         for m = 1:betatau
            term10sum(i) = term10sum(i) + ((rho^(g+(m-1)))/...
                               (pi*k(ct)*I(i)))*(sin(2*pi*k(ct)*I(i)*...
                               (1-((m-1)/betatau)))-sin(((2*pi*k(ct)*I(i))/...
                               betatau)*(max(g-betatau,m-1)-(m-1))));
```

```
            end
         end
         if ct == M(i)/2
            term102kleft(i) = term10sum(i);
         end
      end
      term102klr(i) = term10sum(i);
      term102kright(i) = term102klr(i)-term102kleft(i);
   end
   save term102k term102kleft term102kright term102klr
   exit
```

```
>> term102kleft8

term102kleft8 =

  -6.5584e-07        % CASE: (b-=1,b+=0)

>> term102kright8
                                                          %%%%%%%%%
term102kright8 =                                          % I = 8 %
                                                          %%%%%%%%%
  -6.5584e-07        % CASE: (b-=0,b+=1)

>> term102klr8

term102klr8 =

  -1.3117e-06        % CASE: (b-=1,b+=1)

>>
--------------------------------------------------------------------------------
>> term102kleft9

term102kleft9 =

  -5.2745e-07        % CASE: (b-=1,b+=0)

>> term102kright9
                                                          %%%%%%%%%
term102kright9 =                                          % I = 9 %
                                                          %%%%%%%%%
  -5.2745e-07        % CASE: (b-=0,b+=1)

>> term102klr9

term102klr9 =

  -1.0549e-06        % CASE: (b-=1,b+=1)

>>
--------------------------------------------------------------------------------
>> term102kleft12

term102kleft12 =

  -3.0548e-07        % CASE: (b-=1,b+=0)

>> term102kright12
                                                          %%%%%%%%%%
term102kright12 =                                         % I = 12 %
                                                          %%%%%%%%%%
  -3.0548e-07        % CASE: (b-=0,b+=1)
```

```
>> term102klr12

term102klr12 =

  -6.1096e-07          % CASE: (b-=1,b+=1)

>>
--------------------------------------------------------------------------------
>> term102kleft20

term102kleft20 =

  -1.1211e-07          % CASE: (b-=1,b+=0)

>> term102kright20
                                                   %%%%%%%%%%
term102kright20 =                                  % I = 20 %
                                                   %%%%%%%%%%
  -1.1211e-07          % CASE: (b-=0,b+=1)

>> term102klr20

term102klr20 =

  -2.2423e-07          % CASE: (b-=1,b+=1)

>>
--------------------------------------------------------------------------------
```

```
%%%%%%%%%%%%%%%%%%
%     TERM #10   %
% betatau = 2000 %
%%%%%%%%%%%%%%%%%%
```

```
NOTE: THERE WERE SEVERAL MISHAPS IN COMPUTING THIS TERM. POWER OUTAGES,
      MY ACCOUNT BEING SHUT OFF,ETC. IN THE INTERESTS OF BEING ABLE TO
      COMPUTE RESULTS IN A TIMELY MANNER, FOUR COPIES OF THE PROGRAM FOR
      TERM #10, betatau = 2000 WERE MADE. THE I AND M BLOCK WERE MODIFIED
      TO HOLD ONLY ONE VALUE OF I/M, AND THE OUTER LOOP WAS REDUCED TO
      ONE ITERATION. THEN, THE FOUR PROGRAMS WERE RUN ON FOUR SEPARATE
      WORKSTATIONS. IN THE INTERESTS OF BREVITY WE ONLY SHOW THE SINGLE
      PROGRAM FOR COMPUTING THE TERM AS THE FOUR COPIES ARE ESSENTIALLY
      THE EXACT SAME PROGRAM BUT THEY DO ONLY ONE OUTER LOOP EACH.
      THE RESULTS OF THE FOUR PROGRAMS ARE TABULATED HERE USING THE
      A TEXT EDITOR.
```

```
%
%                               THESIS COMPUTER WORK
%
%                               GENERATION OF GRAPHS
%                                      FOR
%                                 betatau = 500
%
%                                             DATE: 19 OCTOBER 1994
%
%     COMMENTS: THIS PROGRAM CONTAINS THE RESULTS OF ALL TEN TERMS OF
%               APPENDIX B RESULTING IN THE PROGRAM PRODUCING A
%               COMPLETE, ACCURATE GRAPH OF THE SYSTEM'S PROBABILITY
%               OF BIT ERROR FOR ALL FOUR VALUES OF I.
%               I = [4 5 8 20]
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
format short e
rho = 0.99;
c = (1-rho)^2;
I = [4 5 8 20];
M = [124 98 60 24];
bitmatrix = [0 0 0; 0 0 1; 0 1 0; 0 1 1; 1 0 0; 1 0 1; 1 1 0; 1 1 1];

term5 = [1.5423e+03 0.9419e+03 0.3307e+03 0.0431e+03
         1.5423e+03 0.9419e+03 0.3307e+03 0.0431e+03
         1.3825e+03 0.9001e+03 0.3576e+03 0.0580e+03];

term8 = [-3.7098e+02 -2.3852e+02 -9.2820e+01 -1.4109e+01
          -3.7098e+02 -2.3852e+02 -9.2820e+01 -1.4109e+01
          -7.4196e+02 -4.7703e+02 -1.8564e+02 -2.8217e+01];

term9 = [125.1608 80.4704 31.3156 4.7599
          125.1608 80.4704 31.3156 4.7599
          250.3216 160.9408 62.6313 9.5199];

term10 = [-2.4375 -1.5672 -0.6099 -0.0927
           -2.4375 -1.5672 -0.6099 -0.0927
           -4.8750 -3.1343 -1.2197 -0.1854];

for ct = 1:4
   for i = 1:8
      for boo = [1 0]
         X1 = boo*c*7.0510e+03;
         X2 = bitmatrix(i,3)*c*959.5662;
         X3 = bitmatrix(i,3)*c*1.2211e+01;
         X4 = bitmatrix(i,3)*c*3.0440e-01;
         if bitmatrix(i,1)==1 & bitmatrix(i,2)==0
            X5 = c*term5(1,ct);
         elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
            X5 = c*term5(2,ct);
         elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
            X5 = c*term5(3,ct);
```

```
          else
             X5 = 0;
          end
          X6 = boo*bitmatrix(i,3)*c*1.8585e+03;
          X7 = boo*bitmatrix(i,3)*c*9.2657e+01;
          if boo == 0
             X8 = 0;
          elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
             X8 = c*term8(1,ct);
          elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
             X8 = c*term8(2,ct);
          elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
             X8 = c*term8(3,ct);
          else
             X8 = 0;
          end
          if bitmatrix(i,3) == 0
             X9 = 0;
          elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
             X9 = c*term9(1,ct);
          elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
             X9 = c*term9(2,ct);
          elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
             X9 = c*term9(3,ct);
          else
             X9 = 0;
          end
          if bitmatrix(i,3) ==0
             X10 = 0;
          elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
             X10 = c*term10(1,ct);
          elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
             X10 = c*term10(2,ct);
          elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
             X10 = c*term10(3,ct);
          else
             X10 = 0;
          end
          INTERSUM = X1+X2+X3+X4+X5+X6+X7+X8+X9+X10;
          if boo==1
             X1BETA(ct,i) = INTERSUM;
          else
             X0BETA(ct,i) = INTERSUM;
          end
       end
    end
X1MIN(ct) = min(X1BETA(ct,:));
X0MAX(ct) = max(X0BETA(ct,:));
VT(ct) = (X0MAX(ct) + X1MIN(ct))/2;
end
RPSQR_TDIVNO_DB = 10:.01:20;
```

```
RPSQR_TDIVNO = 10.^(RPSQR_TDIVNO_DB*0.10);
SINGCHAN = 0.5*erfc(RPSQR_TDIVNO/8^0.5);
for ct = 1:4
   PE(ct,1:1001) = zeros(1,1001);

   for i = 1:8
      PE(ct,:)=PE(ct,:)+0.25*erfc((RPSQR_TDIVNO/2^0.5)*(X1BETA(ct,i)-VT(ct)))...
               +0.25*erfc((RPSQR_TDIVNO/2^0.5)*(VT(ct)-X0BETA(ct,i)));
   end
end
PEFINAL = PE/8;
figure(1)
semilogy(RPSQR_TDIVNO_DB,SINGCHAN,'--',RPSQR_TDIVNO_DB,PEFINAL(1,:),...
         RPSQR_TDIVNO_DB,PEFINAL(2,:),RPSQR_TDIVNO_DB,PEFINAL(3,:),...
         RPSQR_TDIVNO_DB,PEFINAL(4,:))
xlabel('Z (dB)');
ylabel('Pb');
axis([10 17 10^(-15) 1])
```

```
%
%                                  THESIS COMPUTER WORK
%
%                                  GENERATION OF GRAPHS
%                                          FOR
%                                     betatau = 1000
%
%                                                      DATE: 21 OCTOBER 1994
%
%    COMMENTS: THIS GRAPHING PROGRAM DOES NOT CONTAIN THE RESULTS FOR
%              TERM #5, APPENDIX B DUE TO AN INABILITY TO ARRIVE AT A
%              SOLUTION. THERE WERE MEMORY PROBLEMS WHEN A MATRIX SOLUTION
%              WAS TRIED TO SPEED CALCULATION OF THIS TERM, AND THE
%              TIME REQUIRED TO CALCULATE USING SIMPLE LOOPS WAS
%              PROHIBITIVE. THUS, ON THE RESULTING GRAPH PRODUCED
%              ONLY THE I = 20 TRACE IS ACCURATE IN PREDICTING
%              THE SYSTEM'S PROBABLILITY OF BIT ERROR. THIS IS BECAUSE
%              AT I = 20, THE EFFECTS OF ADJACENT CHANNEL INTERFERENCE (ACI)
%              ARE NEGLIGABLE.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
rho = 0.99;
c = (1-rho)^2;
I = [5 6 9 20];
M = [198 164 110 48];
bitmatrix = [0 0 0; 0 0 1; 0 1 0; 0 1 1; 1 0 0; 1 0 1; 1 1 0; 1 1 1];

term8 = [-464.3344 -327.7899 -148.2912 -29.7545
         -464.3344 -327.7899 -148.2912 -29.7545
         -928.6688 -655.5798 -296.5824 -59.5090];

term9 = [155.3047 109.6350 49.5986 9.9519
         155.3047 109.6350 49.5986 9.9519
         310.6094 219.2701 99.1971 19.9038];

term10 = [-0.0200 -0.0142 -0.0064 -0.0013
          -0.0200 -0.0142 -0.0064 -0.0013
          -0.0401 -0.0283 -0.0128 -0.0026];

for ct = 1:4
   for i = 1:8
      for boo = [1 0]
         X1 = boo*c*8.5126e+03;
         X2 = bitmatrix(i,3)*c*492.4271;
         X3 = bitmatrix(i,3)*c*4.2917e-02;
         X4 = bitmatrix(i,3)*c*1.5865e-05;
         % NOTE THAT THE RESULTS FOR TERM #5, APPENDIX B ARE
         % MISSING. SEE THE COMMENTS IN THE TITLE SECTION
         % ABOVE.
         X6 = boo*bitmatrix(i,3)*c*9.9411e+02;
         X7 = boo*bitmatrix(i,3)*c*7.3500e-01;
```

```
         if boo == 0
            X8 = 0;
         elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
            X8 = c*term8(1,ct);
         elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
            X8 = c*term8(2,ct);
         elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
            X8 = c*term8(3,ct);
         else
            X8 = 0;
         end
         if bitmatrix(i,3) == 0
            X9 = 0;
         elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
            X9 = c*term9(1,ct);
         elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
            X9 = c*term9(2,ct);
         elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
            X9 = c*term9(3,ct);
         else
            X9 = 0;
         end
         if bitmatrix(i,3) ==0
            X10 = 0;
         elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
            X10 = c*term10(1,ct);
         elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
            X10 = c*term10(2,ct);
         elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
            X10 = c*term10(3,ct);
         else
            X10 = 0;
         end
         INTERSUM = X1+X2+X3+X4+X6+X7+X8+X9+X10;
         if boo==1
            X1BETA(ct,i) = INTERSUM;
         else
            X0BETA(ct,i) = INTERSUM;
         end
      end
   end
X1MIN(ct) = min(X1BETA(ct,:));
X0MAX(ct) = max(X0BETA(ct,:));
VT(ct) = (X0MAX(ct) + X1MIN(ct))/2;
end
RPSQR_TDIVNO_DB = 10:.01:20;
RPSQR_TDIVNO = 10.^(RPSQR_TDIVNO_DB*0.10);
SINGCHAN = 0.5*erfc(RPSQR_TDIVNO/8^0.5);
for ct = 1:4
   PE(ct,1:1001) = zeros(1,1001);
   for i = 1:8
```

```
      PE(ct,:)=PE(ct,:)+0.25*erfc((RPSQR_TDIVNO/2^0.5)*(X1BETA(ct,i)-VT(ct)))...
               +0.25*erfc((RPSQR_TDIVNO/2^0.5)*(VT(ct)-X0BETA(ct,i)));
   end
end
PEFINAL = PE/8;
figure(1)
semilogy(RPSQR_TDIVNO_DB,SINGCHAN,'--',RPSQR_TDIVNO_DB,PEFINAL(1,:),...
         RPSQR_TDIVNO_DB,PEFINAL(2,:),RPSQR_TDIVNO_DB,PEFINAL(3,:),...
         RPSQR_TDIVNO_DB,PEFINAL(4,:))
xlabel('Z (dB)');
ylabel('Pb');
axis([10 17 10^(-15) 1])
```

```
%                                THESIS COMPUTER WORK
%
%                                GENERATION OF GRAPHS
%                                       FOR
%                                  betatau = 1500
%
%                                                     DATE: 25 OCTOBER 1994
%
%    COMMENTS: THIS GRAPHING PROGRAM DOES NOT CONTAIN THE RESULTS FOR
%              TERM #5, APPENDIX B DUE TO AN INABILITY TO ARRIVE AT A
%              SOLUTION. THERE WERE MEMORY PROBLEMS WHEN A MATRIX SOLUTION
%              WAS TRIED TO SPEED CALCULATION OF THIS TERM, AND THE
%              TIME REQUIRED TO CALCULATE USING SIMPLE LOOPS WAS
%              PROHIBITIVE. THUS, ON THE RESULTING GRAPH PRODUCED
%              ONLY THE I = 20 TRACE IS ACCURATE IN PREDICTING
%              THE SYSTEM'S PROBABLILITY OF BIT ERROR. THIS IS BECAUSE
%              AT I = 20, THE EFFECTS OF ADJACENT CHANNEL INTERFERENCE (ACI)
%              ARE NEGLIGABLE.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
rho = 0.99;
c = (1-rho)^2;
I = [7 9 12 20];
M = [212 164 124 74];
bitmatrix = [0 0 0; 0 0 1; 0 1 0; 0 1 1; 1 0 0; 1 0 1; 1 1 0; 1 1 1];

term8 = [-3.5253e+02 -2.1854e+02 -1.2475e+02 -4.5096e+01
         -3.5253e+02 -2.1854e+02 -1.2475e+02 -4.5096e+01
         -7.0505e+02 -4.3708e+02 -2.4950e+02 -9.0192e+01];

term9 = [117.9019 73.0900 41.7221 15.0822
         117.9019 73.0900 41.7221 15.0822
         235.8037 146.1800 83.4441 30.1644];

term10 = [-1.0000e-04 -0.6199e-04 -0.3539e-04 -0.1279e-04
          -1.0000e-04 -0.6199e-04 -0.3539e-04 -0.1279e-04
          -0.2000e-03 -0.1240e-03 -0.0708e-03 -0.0256e-03];

for ct = 1:4
   for i = 1:8
      for boo = [1 0]
         X1 = boo*c*9.0083e+03;
         X2 = bitmatrix(i,3)*c*328.3413;
         X3 = bitmatrix(i,3)*c*1.8815e-04;
         X4 = bitmatrix(i,3)*c*7.2482e-10;
         % NOTE THAT THE RESULTS FOR TERM #5, APPENDIX B ARE
         % MISSING. SEE THE COMMENTS IN THE TITLE SECTION
         % ABOVE.
         X6 = boo*bitmatrix(i,3)*c*6.6331e+02;
         X7 = boo*bitmatrix(i,3)*c*5.1105e-03;
         if boo == 0
```

```
            X8 = 0;
         elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
            X8 = c*term8(1,ct);
         elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
            X8 = c*term8(2,ct);
         elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
            X8 = c*term8(3,ct);
         else
            X8 = 0;
         end
         if bitmatrix(i,3) == 0
            X9 = 0;
         elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
            X9 = c*term9(1,ct);
         elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
            X9 = c*term9(2,ct);
         elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
            X9 = c*term9(3,ct);
         else
            X9 = 0;
         end
         if bitmatrix(i,3) ==0
            X10 = 0;
         elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
            X10 = c*term10(1,ct);
         elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
            X10 = c*term10(2,ct);
         elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
            X10 = c*term10(3,ct);
         else
            X10 = 0;
         end
         INTERSUM = X1+X2+X3+X4+X6+X7+X8+X9+X10;
         if boo==1
            X1BETA(ct,i) = INTERSUM;
         else
            X0BETA(ct,i) = INTERSUM;
         end
      end
   end
X1MIN(ct) = min(X1BETA(ct,:));
X0MAX(ct) = max(X0BETA(ct,:));
VT(ct) = (X0MAX(ct) + X1MIN(ct))/2;
end
RPSQR_TDIVNO_DB = 10:.01:20;
RPSQR_TDIVNO = 10.^(RPSQR_TDIVNO_DB*0.10);
SINGCHAN = 0.5*erfc(RPSQR_TDIVNO/8^0.5);
for ct = 1:4
   PE(ct,1:1001) = zeros(1,1001);
   for i = 1:8
      PE(ct,:)=PE(ct,:)+0.25*erfc((RPSQR_TDIVNO/2^0.5)*(X1BETA(ct,i)-VT(ct)))...
```

```
                +0.25*erfc((RPSQR_TDIVNO/2^0.5)*(VT(ct)-XOBETA(ct,i)));
  end
end
PEFINAL = PE/8;
figure(1)
semilogy(RPSQR_TDIVNO_DB,SINGCHAN,'--',RPSQR_TDIVNO_DB,PEFINAL(1,:),...
         RPSQR_TDIVNO_DB,PEFINAL(2,:),RPSQR_TDIVNO_DB,PEFINAL(3,:),...
         RPSQR_TDIVNO_DB,PEFINAL(4,:))
xlabel('Z (dB)');
ylabel('Pb');
axis([10 17 10^(-15) 1])
```

```
%                               THESIS COMPUTER WORK
%
%                               GENERATION OF GRAPHS
%                                       FOR
%                                 betatau = 2000
%
%                                                      DATE: 15 NOVEMBER 1994
%
%     COMMENTS: THIS GRAPHING PROGRAM DOES NOT CONTAIN THE RESULTS FOR
%               TERM #5, APPENDIX B DUE TO AN INABILITY TO ARRIVE AT A
%               SOLUTION. THERE WERE MEMORY PROBLEMS WHEN A MATRIX SOLUTION
%               WAS TRIED TO SPEED CALCULATION OF THIS TERM, AND THE
%               TIME REQUIRED TO CALCULATE USING SIMPLE LOOPS WAS
%               PROHIBITIVE. THUS, ON THE RESULTING GRAPH PRODUCED
%               ONLY THE I = 20 TRACE IS ACCURATE IN PREDICTING
%               THE SYSTEM'S PROBABLILITY OF BIT ERROR. THIS IS BECAUSE
%               AT I = 20, THE EFFECTS OF ADJACENT CHANNEL INTERFERENCE (ACI)
%               ARE NEGLIGABLE.
%
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
rho = 0.99;
c = (1-rho)^2;
I = [8 9 12 20];
M = [248 220 164 98];
bitmatrix = [0 0 0; 0 0 1; 0 1 0; 0 1 1; 1 0 0; 1 0 1; 1 1 0; 1 1 1];

term8 = [-351.8931 -283.0055 -163.9044 -60.1542
         -351.8931 -283.0055 -163.9044 -60.1542
         -703.7863 -566.0110 -327.8088 -120.3084];

term9 = [91.5258 73.0261 41.6758 15.0544
         91.5258 73.0261 41.6758 15.0544
         183.0515 146.0523 83.3516 30.1088];

term10 = [-6.5584e-07 -5.2745e-07 -3.0548e-07 -1.1211e-07
          -6.5584e-07 -5.2745e-07 -3.0548e-07 -1.1211e-07
          -1.3117e-06 -1.0549e-06 -6.1096e-07 -2.2423e-07];

for ct = 1:4
   for i = 1:8
      for boo = [1 0]
         X1 = boo*c*9.2563e+03;
         X2 = bitmatrix(i,3)*c*246.2563;
         X3 = bitmatrix(i,3)*c*9.2720e-07;
         X4 = bitmatrix(i,3)*c*3.2152e-14;
         % NOTE THAT THE RESULTS FOR TERM #5, APPENDIX B ARE
         % MISSING. SEE THE COMMENTS IN THE TITLE SECTION
         % ABOVE.
         X6 = boo*bitmatrix(i,3)*c*4.9749e+02;
         X7 = boo*bitmatrix(i,3)*c*3.4503e-05;
```

```
         if boo == 0
            X8 = 0;
         elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
            X8 = c*term8(1,ct);
         elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
            X8 = c*term8(2,ct);
         elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
            X8 = c*term8(3,ct);
         else
            X8 = 0;
         end
         if bitmatrix(i,3) == 0
            X9 = 0;
         elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
            X9 = c*term9(1,ct);
         elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
            X9 = c*term9(2,ct);
         elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
            X9 = c*term9(3,ct);
         else
            X9 = 0;
         end
         if bitmatrix(i,3) ==0
            X10 = 0;
         elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
            X10 = c*term10(1,ct);
         elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
            X10 = c*term10(2,ct);
         elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
            X10 = c*term10(3,ct);
         else
            X10 = 0;
         end
         INTERSUM = X1+X2+X3+X4+X6+X7+X8+X9+X10;
         if boo==1
            X1BETA(ct,i) = INTERSUM;
         else
            X0BETA(ct,i) = INTERSUM;
         end
      end
   end
X1MIN(ct) = min(X1BETA(ct,:));
X0MAX(ct) = max(X0BETA(ct,:));
VT(ct) = (X0MAX(ct) + X1MIN(ct))/2;
end
RPSQR_TDIVNO_DB = 10:.01:20;
RPSQR_TDIVNO = 10.^(RPSQR_TDIVNO_DB*0.10);
SINGCHAN = 0.5*erfc(RPSQR_TDIVNO/8^0.5);
for ct = 1:4
   PE(ct,1:1001) = zeros(1,1001);
   for i = 1:8
```

```
      PE(ct,:)=PE(ct,:)+0.25*erfc((RPSQR_TDIVNO/2^0.5)*(X1BETA(ct,i)-VT(ct)))...
               +0.25*erfc((RPSQR_TDIVNO/2^0.5)*(VT(ct)-X0BETA(ct,i)));
   end
end
PEFINAL = PE/8;
figure(1)
semilogy(RPSQR_TDIVNO_DB,SINGCHAN,'--',RPSQR_TDIVNO_DB,PEFINAL(1,:),...
         RPSQR_TDIVNO_DB,PEFINAL(2,:),RPSQR_TDIVNO_DB,PEFINAL(3,:),...
         RPSQR_TDIVNO_DB,PEFINAL(4,:))
xlabel('Z (dB)');
ylabel('Pb');
axis([10 17 10^(-15) 1])
```

REFERENCES

[1] W. M. Hamdy and P. A. Humblet, "Sensitivity Analysis of Direct Detection Optical FDMA Networks with OOK Modulation," *J. Lightwave Technology*, Vol. 11, pp. 783–794, May/June 1993.

[2] T. C. Chou, "A Simple Analytical Model for Dense WDM/OOK Systems," Master's Thesis, Naval Postgraduate School, June 1994.

[3] P. A. Humblet and W. M. Handy, "Crosstalk Analysis and Filter Optimization of Single- and Double-Cavity Fabry-Perot Filters," *IEEE J. on Selected Areas in Communications*, Vol. 8, No. 6, pp. 1095–1107, August 1990.

[4] L. W. Couch, II, *Digital and Analog Communications Systems*, 2nd Ed., New York: MacMillan Publishing Co., 1987.

[5] A. D. Kraus, *Circuit Analysis*, St Paul, MN: West Publishing Co., 1991.

[6] R. E. Walpole and R. H. Meyers, *Probability and Statistics for Engineers and Scientists*, 4th Ed., New York: Macmillan Publishing Co., 1989.

[7] J. P. Powers, *An Introduction to Fiber Optic Systems*, Boston, MA: Aksen Associates, Inc., Publishers, 1993.

[8] G. R. Fowles, *Introduction to Modern Optics*, 2nd Ed., New York: Dover Publications, Inc., 1989.

[9] A. Leon-Garcia, *Probability and Random Processes for Electrical Engineering*, New York: Addison-Wesley Publishing Co., 1989.

[10] A. V. Oppenheim and A. S. Willsky, *Signals and Systems*, Englewood Cliffs, NJ: Prentice-Hall, Inc., 1983.

[11] J. G. Proakis, *Digital Communications*, 2nd Ed., New York: McGraw-Hill, Inc., 1989.

[12] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, 3rd Ed., New York: McGraw-Hill, Inc., 1991.

[13] B. Sklar, *Digital Communications, Fundamentals and Applications*, Englewood Cliffs, NJ: PTR Prentice-Hall, Inc., 1988.

[14] S. Haykin, *An Introduction to Analog and Digital Communications*, New York: John Wiley & Sons, 1989.

[15] D. Halliday and R. Resnick, *Fundamentals of Physics*, 3rd Ed., New York: John Wiley & Sons, 1988.

[16] R. L. Finney and G. B. Thomas, Jr., *Calculus*, New York: Addison-Wesley Publishing Co., 1990.

[17] W. M. Hamdy, "Crosstalk in Optical Direct Detection FDMA Networks," Ph.D. Dissertation, Massachusetts Institute of Technology, 1991, LIDS-TH-2053.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Information Center Cameron Station Alexandria, VA 22304-6145	2
2.	Library Code 52 Naval Postgraduate School Monterey, CA 93943-5101	2
3.	Chairman, Code EC Department of Electrical and Computer Engineering Naval Postgraduate School Monterey, CA 93943-5121	1
4.	Professor Tri T. Ha, Code EC/Ha Department of Electrical and Computer Engineering Naval Postgraduate School Monterey, CA 93943-5121	3
5.	Professor Randy L. Borchardt, Code EC/Bt Department of Electrical and Computer Engineering Naval Postgraduate School Monterey, CA 93943-5121	2
6.	Professor John P. Powers, Code EC/Po Department of Electrical and Computer Engineering Naval Postgraduate School Monterey, CA 93943-5121	2
7.	Army Research Laboratory 2800 Powder Mill Road Adelphi, MD 20783-1145	1
8.	CPT John A. Studer c/o Mr. and Mrs. M. P. Studer 702 Mercer Avenue Ojai, CA 93023	2